CSC 2411H - Assignment 2

Due Feb 21, 2005

1. (a) Show that if a variable x_j leaves the basis, it cannot return to it in the next iteration of the simplex algorithm (regardless of the pivot rule) . Deduce that there are no cycles of size two in the algorithm.

   (b) Give an example showing that a variable x_j can join the basis and leave it in the next iteration.

2. Consider the LP

$$\begin{aligned} & \min\langle x, c\rangle \\ \text{s.t.} & \\ & \textstyle\sum_i x_i = 1 \\ & x \geq 0 \end{aligned}$$

   (a) Find an objective vector c and a pivot rule (you may use one of the rules mentioned in class) so that simplex algorithm that starts at a vertex of your choice visits *all* vertices of the solution polyhedron.

   (b) Why isn't this example a simple alternative to the Klee and Minty's cube, that shows an exponential time lower bound for the simplex algorithm?

3. Let $p_1, p_2, \ldots, p_r$ and $q_1, q_2, \ldots, q_s$ be points in $\mathbb{R}^n$, and let $P = \text{conv}(p_1, p_2, \ldots, p_r)$ and $Q = \text{conv}(q_1, q_2, \ldots, q_r)$ be their convex hulls.

   Our goal is to find a strict separating hyperplane between P and Q. In other words a hyperplane H with corresponding halfspaces H^+ and H^- so that $P \subset H^- \setminus H$ and $Q \subset H^+ \setminus H$.

   (a) Use LP-duality to decide under what condition do P and Q have a strict separating hyperplanes.

(b) Assume that P and Q are 3-D polytopes. Give a *linear* time algorithm that finds a strict separating hyperplane if there is one, and otherwise announces that there isn't any.

4. Use Strong Duality Theorem Duality for (when the primal is in) standard form, or Farkas Lemma to show the following.

   (a) Strong Duality Theorem Duality theorem in Canonical form.

   (b) Either $Ax > 0$ has a solution, or $yA = 0, y \geq 0, y \neq 0$ has a solution, but not both.

5. Theorem: if $K_1, K_2, \ldots, K_t$ are convex bodies in $\mathbb{R}^n$ such that every $n+1$ of them intersect, then there is a point that belongs to all of them. Prove the theorem for the special case where the K_i are halfspaces.

6. Check for yourself (don't submit)

   (a) Prove that the dual of the dual is the primal. Your initial LP should be in general form (minimization, and involving equality and inequality constraints, nonnegative and free variables).

   (b) We know that if a primal LP is unbounded then its dual is infeasible. Is the converse true? In other words, is it impossible for a primal and dual problems to be both infeasible? Prove or give a counter example.