

UNIVERSITY OF CALIFORNIA
Department of Electrical Engineering
and Computer Sciences
Computer Science Division

CS61B
Summer

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Homework #4

Due: Tuesday, July 18, 2000

Create a directory to hold your answers to this homework set. You will find templates for some problems in `~/cs61b/hw/hw4`; please use them. To submit your homework, use the command `'submit hw4'` from within that directory. Where the problems specify file names to use for your solution, use exactly those names. Put your answers to non-programming questions in a file called `hw4.txt`. (For typing purposes, feel free to use “Omega” and “Theta” in place of Ω and Θ).

1. Determine the running time of the following two program segments:

```
\\Fragment 1
for(int i = 0; i < n; i++)
  for (int j = 0; j < n * n; j++)
    for (int k = 0; k < j; k++)
      sum++;
```

```
\\Fragment 2
for(int i = 0; i < n; i++)
  for (int j = 0; j < i * i; j++)
    if (j % i == 0)
      for (int k = 0; k < j; k++)
        sum++;
```

2. Show each of the following *false* by exhibiting a counterexample. Assume that f and g are any real-valued functions.

- a. $O(f(x) \cdot s(x)) = o(f(x))$, assuming $\lim_{x \rightarrow \infty} s(x) = 0$.
- b. If $f(x) \in O(x^3)$ and $g(x) \in O(x)$ then $f(x)/g(x) \in O(x^2)$.
- c. If $f(x) \in \Omega(x)$ and $g(x) \in \Omega(x)$ then $f(x) + g(x) \in \Omega(x)$.
- d. If $f(100) = 1000$ and $f(1000) = 1000000$ then f cannot be $O(1)$.
- e. If $f_1(x), f_2(x), \dots$ are a bunch of functions that are all in $\Omega(1)$, then the sum

$$F(x) = \sum_{1 \leq i \leq N} f_i(x) \in \Omega(N).$$

3. Demonstrate the following (that is, demonstrate appropriate constants to plug into the definitions of $O(\cdot)$, $\Omega(\cdot)$, or $\Theta(\cdot)$).

- a. $n! \in \Omega(2^n)$
- b. $(\log n)^K \in O(n)$ for any K .
- c. $n^3 \in \Theta(n^3 + 2n^2)$
- d. $\sum_{1 < i \leq n} 1/i \in O(\log n)$
- e. $O(|f(x)| + |g(x)|) = O(\max(|f(x)|, |g(x)|))$.

4. Fill in the following, using *constant* extra space (therefore, recursion, auxiliary linked lists, auxiliary arrays, **Vectors**, **Stacks**, and the like are illegal!). Put your answer in file `ReversePrinter.java`:

```
class ReversePrinter {
    /** Print the string form of the items in L in reverse on
     * str, one to a line. Does not permanently alter L. */
    public static void reverse (List L, PrintStream str) { // FILL IN
    }
}
```

5. The last element of a linked list does not have to be null. It can point to another element of the same list, causing a phenomenon known as circular lists. In particular, the last element can point to itself (known as knot) or to some previous element (known as loop). Fill in the following, using *constant* extra space. Put your answer in file `LoopTester.java`:

```
class LoopTester {
    /** True iff L is a loop. */
    public static boolean isLoop (List L) { // FILL IN
    }
}
```

6. (From the previous homework) Given an array with positive integers, we wish to replace each integer with the closest to the right larger one, if no such element exists it should be replaced with 0. Given the array `{ 1, 5, 3, 7, 2, 3 }` we should get `{ 5, 7, 7, 0, 3, 0 }`. Implement this function so that it takes time linear to the length of the array.

```
class Shadow {
    /** Changes each number to teh closest to the right larger one
     */
    static int[] shadow(int[] x) { /* Fill in */
    }
}
```