

Assignment 3

CSC373 Winter 2010

Due Date: April 1st, Noon, BA2220 Drop Box 8

Problem 1

DPV 9.5

Problem 2

Consider the Bin Packing problem: given positive integer B , and a list of integers $A = [a_1, \dots, a_n]$ (where $0 < a_i \leq B$), try to partition A into as few cells as possible such that for each cell of the partition, the sum of the elements in the cell is at most B .

The FirstFit algorithm takes each element of A in turn (note that we do NOT sort the objects by size) and places it into the first bin that can accommodate it.

For example, suppose $B = 10$ and $A = [2, 2, 7, 8, 3, 6, 3, 2, 6]$. FirstFit would use 5 bins and fill them as follows: Bin(1) contains 2, 2, 3 and 3; Bin(2) contains 7 and 2; Bin(3) contains 8; Bin(4) contains 6; and Bin(5) contains 6.

$$\text{Let } S = \sum_{i=1}^n a_i$$

.

- (a) Argue that the optimum number of bins required is at least $\lceil S/B \rceil$.
- (b) Argue that the FirstFit algorithm leaves at most one bin less than half full.
- (c) Prove that the number of bins used by FirstFit is never more than $\lceil 2S/B \rceil$.
- (d) Prove an approximation ratio of 2 for the FirstFit algorithm. That is, prove that for any input A and integer B , FF uses at most twice the number of bins that an optimal algorithm would use.

Problem 3

DPV 9.7

Problem 4

DPV 7.16 (I like <http://vinci.inesc.pt/lp/>, once you get the hang of the syntax).