# Tutorial 7 - Functional Depedency 

## CSC343 - Introduction to Databases <br> Fall 2008

TA: LeíJíang

## Anomalie

- Redundancy leads to anomalies. Consider the relation schema: Rents(CustomerID, Title, Price, Rating, Date), and following instance of the schema (which records customer renting movies at some dates). Describe update, deletion and insertion anomalies caused by this design.

| CustomerID | Title | Price | Rating | Date |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0001 | Braveheart | 4.20 | PG13 | 2003-01-08 |
| 0001 | The Patriot | 3.30 | R | $2003-01-08$ |
| 0002 | The Patriot | 3.30 | R | $2003-03-03$ |
| 0003 | Ransom | 3.50 | PG13 | $2003-04-01$ |

## Functional Dependency

- FD: given schema $\mathbf{R}(\mathbf{X})$ and non-empty subsets $\mathbf{Y}$ and $\mathbf{Z}$ of the attributes $\mathbf{X}$, we say that there is a functional dependency between $\mathbf{Y}$ and $\mathbf{Z}(\mathbf{Y} \rightarrow \mathbf{Z})$, iff for every relation instance $\mathbf{r}$ of $\mathbf{R}(\mathbf{X})$ and every pair of tuples $\mathbf{t 1}, \mathbf{t} 2$ of $\mathbf{r}$, if $\mathbf{t} 1 . \mathrm{Y}=\mathrm{t} 2 . \mathrm{Y}$, then $\mathbf{t 1 . Z}=\mathbf{t 2} . \mathrm{Z}$
- ..., or, ... the values of $\mathbf{Y}$ uniquely determines the values of $\mathbf{Z}$ in all possible instances of R. (FD on non-keys causes redundancy)
- Examples:
- A film has a uniaue title. rental brice and distributor.

FilmID $\rightarrow$ Title, RentalPrice, Distributor

- The customerID uniquely identifies the customer and his/her address

CustomerID $\rightarrow$ Name, Street, City, State

- On any particular day, a film tape can be rented to at most one customer.

Date, FilmID, TapeNum $\rightarrow$ CustomerID

- A performer can have only one role in a particular movie.

PerformerID, FilmID $\rightarrow$ Role

## Armstrong's Axioms

- Armstrong
- Reflexivity: $Y \subseteq X \quad \mid-X \rightarrow Y$
- Augmentation: $X \rightarrow Y$ |- $X Z \rightarrow Y Z$
- Transitivity: $X \rightarrow Y, Y \rightarrow Z$ |- $X \rightarrow Z$
- Derived Rules
- Decomposition: $X \rightarrow Y Z$ |- $X \rightarrow Y, X \rightarrow Z$
- Union: $X \rightarrow Y$ and $X \rightarrow Z$ |- $X \rightarrow Y Z$


## Armstrong's Axioms

- Proof following rules using Armstrong's Axioms - Left Augmentation

$$
X \rightarrow Y \quad \mid-X Z \rightarrow Y
$$

- Pseudotransitivity

$$
X \rightarrow Y, Y Z \rightarrow W \quad \mid-\quad X Z->W
$$

- Addition
$X \rightarrow Y, Z \rightarrow W \mid-X Z \rightarrow Y W$


## Armstrong's Axioms

- Let $\boldsymbol{F}$ be the following set of functional dependencies: $\{A B \rightarrow C D, B \rightarrow D E, C \rightarrow F$, $E \rightarrow G, A \rightarrow B\}$. Use Armstrong's axioms to show that $\{A \rightarrow F G\}$ is logically implied by $F$



## FD Closure

- A FD $\mathbf{f}: \mathbf{Y} \rightarrow \mathbf{Z}$ on schema $\mathbf{R}(\mathbf{X})$ is a constraint on all allowable instances of $\mathbf{R}$
- $\mathbf{F}$ entails $\mathbf{f}$ if every instance of $R$ that satisfies $\mathbf{F}$ also satisfies $\mathbf{f}$.
- The closure of $\mathbf{F}$, denoted $\mathbf{F}^{+}$, is the set of all FDs entailed by $\mathbf{F}$.
- Given a set of FDs F, the attribute closure of a set of attributes $X$ is $X^{+}{ }_{F}=$ set of all attributes $A$ such that $X \rightarrow A$ (entailed by $F$ )
closure $:=X ; \quad / /$ since $X \subseteq X_{F}{ }_{F}$
repeat
old := closure;
if there is an FD $Z \rightarrow V$ in $F$ such that
$Z \subseteq$ closure and $V \nsubseteq$ closure
then closure $:=$ closure $\cup V$
until old = closure
- If $T \subseteq$ closure then $X \rightarrow T$ is entailed by $F$


## FD Closure

- Given $\boldsymbol{R}=A B C D$ and $\boldsymbol{F}=\{A \rightarrow B, A \rightarrow C, C D$ $\rightarrow A\}$. Compute $F^{+}$.
- Solution:
$-A^{+}{ }_{F}=\{A B C\}$
$-B^{+}{ }_{F}=\{B\}$
- ...
$-A B^{+}{ }_{F}=\{A B C\}$
$-A C^{+}{ }_{F}=\{A B C\}$
- ...
$-A B C^{+}{ }_{F}=\ldots$


## FD Closure

- (Previous final question) Given $\boldsymbol{R}=A B C D E G H$ and $F=\{A \rightarrow D E, C \rightarrow A D H, B H \rightarrow G E, A B H \rightarrow$ $C, B G H \rightarrow C\}$. Compute $X^{+}{ }_{F}$ for sets of attributes $X$ such that $X$ appears on the left hand side of a FD in F.
- Solution:
$-A^{+}{ }_{F}=\{A D E\}$
$-C^{+}{ }_{F}=\{A C D E H\}$
$-B H^{+}{ }_{F}=\{B C E G H\}$
$-A B H^{+}{ }_{F}=\{A B C D E G H\}$
$-B G H^{+}{ }_{F}=\{A B C D E G H\}$
- Use Armstrong's Axioms to prove each FD!


## Key

- Recall a key is a minimal superkey, where superkey of a schema $\mathbf{R}$ is a set of attributes in $\mathbf{R}$ that functionally determines all attributes in $\mathbf{R}$.
- Consider the relation schema $\mathbf{R}(A, B, C, D)$ with FDs: $A \rightarrow C$ and $B \rightarrow D$. Is $\{A, B\}$ a key for $\mathbf{R}$ ?

Fact: $\{A, B\}$ is a superkey.
Indeed from Armstrong's Axioms we can infer:

$$
A \rightarrow C \Rightarrow A B \rightarrow \mathrm{ABC} \text { (augmentation by } \mathrm{AB})
$$

$B \rightarrow D \Rightarrow A B C \rightarrow \mathrm{ABCD}$ (augmentation by ABC )
We obtain $A B \rightarrow A B C D$ (transitivity)
$\{A, B\}$ is a candidate key (minimal). We must show that neither $\{A\}$ nor $\{B\}$ alone are candidate keys.
Neither $\{A\}$ nor $\{B\}$ are superkeys since $\{A\}^{+}=\{A, C\},\{B\}^{+}=\{B, D\}$
(But $\{A, B\}^{+}=\{A, B, C, D\}$ )

## key

- Consider a schema $\mathbf{R}=\{S, T, V, C, P, D\}$ and $F=\{S \rightarrow T, V \rightarrow S C, S D \rightarrow P\}$. Find keys for $\mathbf{R}$.
- Solution
- $\quad V$ and $D$ do not appear on the right side, they must be in the key.
- $\mathrm{VD}^{+}{ }_{\mathrm{F}}=\{S T V C P D\}$. So VD is the only key.
- Now proof: $F I=V D \rightarrow$ STVCPD using Armstrong's Axioms Solution

| (1) $V \rightarrow S C$ | \# given |
| :--- | :--- |
| (2) $V D \rightarrow S C D$ | \# aug. (1) |
| (3) $V D \rightarrow V S C D$ | \# aug. (2) |
| (4) $S \rightarrow T$ | \# given |
| (5) $S C \rightarrow T C$ | \# aug. (4) |
| (6) $V \rightarrow T C$ | \# (1), (5) |
| (7) $V D \rightarrow T C D$ | \# (6) aug. |
| (8) $S D \rightarrow P$ | \# given |
| (9) $V D \rightarrow S D$ | \# decomp. (2) |
| (10) $V D \rightarrow P$ | \# trans. (8), (9) |
| (11) $V D \rightarrow S T V C P D$ | \# union (3),(7),(10) |

## All together

- Consider a relation with schema $R(A, B, C, D)$ and $F D$ 's $F=\{A B \rightarrow C, C \rightarrow D, D \rightarrow A\}$.
- a) Compute $\mathrm{F}^{+}$; You can restrict yourself to non-trivial FD's with single attributes on the right side.

Solution:
$A^{+}{ }^{+}=\{A\}$
$B^{+}{ }_{F}=\{B\}$
$\mathrm{C}^{+}{ }^{\mathrm{F}}=\{\mathrm{ACD}\} \quad$ \# add $\mathrm{C} \rightarrow \mathrm{A}$ to $\mathrm{F}^{+}$
$\mathrm{D}^{+}$ $D^{+}{ }^{+}=\{A D\}$
$\mathrm{AB}^{+} \mathrm{AC}^{+}=\{\mathrm{ABCD}\}$
${A C^{+}}^{+}=\{A C D\}$
$\mathrm{BC}^{+}{ }^{\mathrm{F}}=\{\mathrm{ABCD}$
$\mathrm{BC}^{+{ }^{\mathrm{F}}}=\{\mathrm{ABCD}\}$ $\mathrm{CD}^{+}{ }^{\mathrm{F}}=\{\mathrm{ACD}$, $\mathrm{ABC}^{+}{ }^{-}=\left\{\mathrm{ABCD}^{-1}\right\}$ $A_{A B D}{ }^{+}=\{A B C D\}$ $\mathrm{ABD}^{\mathrm{ACD}^{+}}{ }^{+}=\{\mathrm{ABCD}\}$ $\mathrm{ACDD}^{+}=\{\mathrm{ACD}\}$
$\mathrm{BCD}^{+}=\{\mathrm{ABCD}\}$ $\mathrm{ABCD}_{\mathrm{F}}^{+}=\{\mathrm{ABCD}\}$

$$
\begin{aligned}
& \text { \# add } \mathrm{AB} \rightarrow \mathrm{D} \text { to } \mathrm{F}^{+} \\
& \# \text { add } \mathrm{AC} \rightarrow \mathrm{D} \text { to } \mathrm{F}^{+}
\end{aligned}
$$

\# add $\mathrm{BC} \rightarrow \mathrm{A}, \mathrm{BC} \rightarrow \mathrm{D}$ to $\mathrm{F}^{+}$
$\#$ add $\mathrm{BD} \rightarrow \mathrm{A}, \mathrm{BD} \rightarrow \mathrm{C}$ to $\mathrm{F}^{+}$
$\#$ add $\mathrm{CD} \rightarrow \mathrm{A}$ to $\mathrm{F}^{+}$
$\#$ add $A B C \rightarrow D$ to $\mathrm{F}^{+}$
$\#$ add $A B D \rightarrow C$ to $F^{+}$
$\#$ add $B C D \rightarrow A$ to $\mathrm{F}^{+}$
$\mathrm{F}^{+}=\mathrm{F} \cup\{\mathrm{C} \rightarrow \mathrm{A}, \mathrm{AB} \rightarrow \mathrm{D}, \mathrm{AC} \rightarrow \mathrm{D}, \mathrm{BC} \rightarrow \mathrm{A}, \mathrm{BC} \rightarrow \mathrm{D}, \mathrm{BD} \rightarrow \mathrm{A}, \mathrm{BD} \rightarrow \mathrm{C}, \mathrm{CD} \rightarrow \mathrm{A}, \mathrm{ABC} \rightarrow \mathrm{D}$, $A B D \rightarrow C, B C D \rightarrow A\}+\{$ trivial $F D s\}$.

## All together

- What are all the keys of $R$ ?


## Solution

- Superkeys are sets of attributes whose closures are all attributes: $A B, B C, B D, A B C, A B D, B C D$, ABCD
- Keys are minimal superkeys: $A B, B C, B D$

