Computer Science CSC478/2412 Spring 1997
St. George Campus University of Toronto

Homework Assignment #3
Due: Friday, 7 March 1997
Mark: 15%

1. This exercise shows how to compute bivariate interpolation (see Lipson, page 288).

a. Devise an algorithm for computing $U(x, y) \in F[x, y]$ having degree $< n$ in y, such that

$$U(x, \alpha_k) = \beta_k(x), \qquad k = 0, 1, \ldots, n-1,$$

for prescribed $\alpha_k \in F$, α_k pairwise distinct, and $\beta_k(x) \in F[x]$. Show that $U(x, y)$ is unique.

b. If each $\beta_k(x)$ has degree $< m$, derive a bound on the time $T(m, n)$ required to compute $U(x, y)$, assuming that operations in F have constant time.

c. Compute $U(x, y) \in \mathbb{Z}_7[x, y]$ such that

$$U(x, 0) = x^3 + 5, \qquad U(x, 1) = x^3 + 4x^2 + 6, \qquad U(x, 2) = x^3 + 2.$$

2. Let $T(n)$ denote the time to compute the product of two $[n \times n]$ matrices using recursion but without Strassen's method, and $S(n)$ denote the time to compute the product of two $[n \times n]$ matrices using Strassen's algorithm. (See problem 4.13, page 146 in the textbook.) Then, for a constant C and n a power of 2, we can write

$$\begin{array}{rclrcl} T(1) & = & 1 & S(1) & = & 1 \\ T(n) & = & 8 \cdot T\left(\frac{n}{2}\right) + 4C\left(\frac{n}{2}\right)^2 & S(n) & = & 7 \cdot S\left(\frac{n}{2}\right) + 18C\left(\frac{n}{2}\right)^2, \end{array}$$

a. Solve both recurrences when $n = 2^k$. Give upper and lower bounds when $2^{k-1} < n \leq 2^k$. What conclusions can be drawn from these studies?

b. What are the memory requirements of Strassen's algorithm?

3. Let $z_1 = a + ib$ and $z_2 = c + id$, $a, b, c, d \in \mathbb{R}$, be two complex numbers. The cost of multiplying $z_1 z_2$ using the method given by the definition of the product of two complex numbers is 4 real products, and 2 real additions. Let us assume that products are more expensive than sums. Therefore, we want to reduce the number of real products.

a. Derive an algorithm for computing the multiplication of two complex numbers with 3 products of real numbers.

b. Derive an algorithm for computing the multiplication of three complex numbers with 6 products of real numbers.

c. Derive an algorithm for computing the multiplication of n complex numbers. What is the number of products of real numbers needed?

4. Problem 4.14, page 147 of the textbook. First, do it by hand. Then, implement the FFT algorithm, and check your result.

5. Problem 4.15, page 147 of the textbook.

References

K.O. GEDDES AND S.R. CZAPOR AND G. LABAHN, *Algorithms for Computer Algebra*, Kluwer Academic Publishers, Boston, 1992.

J.D. LIPSON, *Elements of Algebra and Algebraic Computing*, Addison-Wesley, Reading MA, 1981.

V. STRASSEN, "Gaussian elimination is not optimal", *Numerische Mathematik*, **13**, 354-356, 1969.