Learning Deep Structured Models

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R. Urtasun (UofT)

Deep Structured Models

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Current Status of your Field?





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- Part I: Deep learning
- Part II: Deep Structured Models

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Part I: Deep Learning

- Supervised models
- Unsupervised learning (will not talk about this today)
- Generative models (will not talk about this today)

Binary Classification

- Given inputs **x**, and outputs $t \in \{-1, 1\}$
- We want to fit a hyperplane that divides the space into half

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SVMs try to maximize the margin

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Non-linear Predictors

How can we make our classifier more powerful?

Image: A matrix

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Two types of approaches:

• Kernel Trick: Fixed functions and optimize linear parameters on non-linear mapping

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• Deep Learning: Learn parametric non-linear functions

$$y_* = F(\mathbf{x}_*, \mathbf{w})$$

Supervised Learning: Examples



Supervised Deep Learning



Denoising



OCR $2345 \rightarrow 2345 \rightarrow 2345^{\prime}$

Image: A mathematical states and a mathem

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- Wⁱ are the parameters of the *i*-th layer

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• Forward Propagation: compute the output given the input

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- The non-linearity is called a ReLU (rectified linear unit), with $\mathbf{x} \in \mathbb{R}^{D}$, $b^{i} \in \mathbb{R}^{N_{i}}$ the biases and $W^{i} \in \mathbb{R}^{N_{i} \times N_{i-1}}$ the weights

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Alternative Graphical Representation



Relu Interpretation

• Piece-wise linear tiling: mapping is locally linear.



Figure : by M. Ranzato

Interpretation

[1 1 0 0 0 1 0 1 0 0 0 0 1 1 0 1...] motorbike



Interpretation



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$$\mathcal{L}(\mathbf{w}) = rac{1}{N} \sum_{i=1}^{N} \ell(\mathbf{w}, \mathbf{x}^{(i)}, t^{(i)}) + \mathcal{R}(\mathbf{w})$$

with N number of examples, \mathcal{R} a regularizer, and \mathbf{w} contains all parameters
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- The task loss: how we are going to evaluate at test time

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- Probability of class k given input (softmax):

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• Use gradient descent to train the network

$$\min_{\mathbf{w}} \frac{1}{N} \sum_{i} \ell(\mathbf{w}, \mathbf{x}^{(i)}, t^{(i)}) + \mathcal{R}(\mathbf{w})$$

• Efficient computation of the gradients by applying the chain rule

$$\mathbf{x} \longrightarrow \left[\max(0, W_1^T \mathbf{x}) \right]^{\mathbf{h}^1} \left[\max(0, W_2^T \mathbf{h}^1) \right]^{\mathbf{h}^2} \left[W_3^T \mathbf{h}^2 \right]^{\frac{\partial \ell}{\partial \mathbf{y}}} \mathbf{y}$$

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• Note that the **forward pass** is necessary to compute $\frac{\partial \ell}{\partial v}$

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• Need to compute gradient w.r.t. inputs and parameters in each layer

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$$\mathbf{x} \longrightarrow \max(0, W_1^T \mathbf{x}) \overset{\frac{\partial \ell}{\partial \mathbf{h}^1}}{\longleftarrow} \max(0, W_2^T \mathbf{h}^1) \overset{\frac{\partial \ell}{\partial \mathbf{h}^2}}{\longleftarrow} W_3^T \mathbf{h}^2 \overset{\frac{\partial \ell}{\partial y}}{\longleftarrow} \mathbf{y}$$
$$\frac{\partial \ell}{\partial \mathbf{h}^2} = \frac{\partial \ell}{\partial y} \frac{\partial y}{\partial \mathbf{h}^2} = (W^3)^T (p(c|\mathbf{x}) - t)$$

$$\frac{\partial \ell}{\partial W^2} = \frac{\partial \ell}{\partial \mathbf{h}^2} \frac{\partial \mathbf{h}^2}{\partial W^2}$$

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 $\bullet~\mbox{Given}~\frac{\partial\ell}{\partial h^2}$ if we can compute the Jacobian of each module

$$\frac{\partial \ell}{\partial W^2} = \frac{\partial \ell}{\partial \mathbf{h}^2} \frac{\partial \mathbf{h}^2}{\partial W^2}$$
$$\frac{\partial \ell}{\partial \mathbf{h}^1} =$$

• Efficient computation of the gradients by applying the chain rule

$$\mathbf{x} \longrightarrow \max(0, W_1^T \mathbf{x}) \overset{\partial \ell}{\longrightarrow} \max(0, W_2^T \mathbf{h}^1) \overset{\partial \ell}{\longleftarrow} W_3^T \mathbf{h}^2 \overset{\partial \ell}{\longleftarrow} \mathbf{y}$$
$$\frac{\partial \ell}{\partial \mathbf{h}^2} = \frac{\partial \ell}{\partial y} \frac{\partial y}{\partial \mathbf{h}^2} = (W^3)^T (p(c|\mathbf{x}) - t)$$

 $\bullet~\mbox{Given}~\frac{\partial\ell}{\partial h^2}$ if we can compute the Jacobian of each module

$$\frac{\partial \ell}{\partial W^2} = \frac{\partial \ell}{\partial \mathbf{h}^2} \frac{\partial \mathbf{h}^2}{\partial W^2}$$
$$\frac{\partial \ell}{\partial \mathbf{h}^1} = \frac{\partial \ell}{\partial \mathbf{h}^2} \frac{\partial \mathbf{h}^2}{\partial \mathbf{h}^1}$$

Gradient Descent

• Gradient descent is a first order method, where one takes steps proportional to the negative of the gradient of the function at the current point

$$\mathbf{x}_{n+1} = \mathbf{x}_n - \gamma_n
abla F(\mathbf{x}_n)$$

• Example:
$$f(x) = x^4 - 3x^3 + 2$$

```
x_old = 0
x_new = 6 # The algorithm starts at x=6
gamma = 0.01 # step size
precision = 0.00001

def f_derivative(x):
    return 4 * x**3 - 9 * x**2

while abs(x_new - x_old) > precision:
    x_old = x_new
    x_new = x_old - gamma * f_derivative(x_old)
print("Local minimum occurs at", x_new)
```



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$$\min_{\mathbf{w}} \frac{1}{N} \sum_{i=1}^{N} \ell(\mathbf{w}, \mathbf{x}^{(i)}, t^{(i)}) + \mathcal{R}(\mathbf{w})$$

$$\min_{\mathbf{w}} \frac{1}{N} \sum_{i=1}^{N} \ell(\mathbf{w}, \mathbf{x}^{(i)}, t^{(i)}) + \mathcal{R}(\mathbf{w})$$

• We need to compute at each iteration

$$\mathbf{w}_{n+1} = \mathbf{w}_n - \gamma_n \nabla L(\mathbf{w}_n)$$

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• Use the backward pass to compute $\nabla L(\mathbf{w}_n)$ efficiently

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$$\mathbf{w}_{n+1} = \mathbf{w}_n - \gamma_n \nabla L(\mathbf{w}_n)$$

- Use the backward pass to compute $\nabla L(\mathbf{w}_n)$ efficiently
- Recall that the backward pass requires the forward pass first

Toy Code (Matlab): Neural Net Trainer

```
% F-PROP
for i = 1 : nr_layers - 1
  [h{i} jac{i}] = nonlinearity(W{i} * h{i-1} + b{i});
end
h{nr_layers-1} = W{nr_layers-1} * h{nr_layers-2} + b{nr_layers-1};
prediction = softmax(h{1-1});
```

```
% CROSS ENTROPY LOSS
loss = - sum(sum(log(prediction) .* target)) / batch_size;
```

```
% B-pROP
dh{l-1} = prediction - target;
for i = nr_layers - 1 : -1 : 1
Wgrad{i} = dh{i} * h{i-1}';
bgrad{i} = sum(dh{i}, 2);
dh{i-1} = (W{i}' * dh{i}) .* jac{i-1};
end
```

```
% UPDATE
for i = 1 : nr_layers - 1
W{i} = W{i} - (lr / batch_size) * Wgrad{i};
b{i} = b{i} - (lr / batch_size) * bgrad{i};
end
```

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$$\min_{\mathbf{w}} \frac{1}{N} \sum_{i=1}^{N} \ell(\mathbf{w}, \mathbf{x}^{(i)}, t^{(i)}) + \mathcal{R}(\mathbf{w})$$

• We need to compute at each iteration

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with

$$abla L(\mathbf{w}_n) = rac{1}{N} \sum_{i=1}^N \nabla \ell(\mathbf{w}, \mathbf{x}^{(i)}, t^{(i)}) + \nabla \mathcal{R}(\mathbf{w})$$

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- Too expensive when having millions of examples
- Instead approximate the gradient with a mini-batch (subset of examples)

$$\frac{1}{N}\sum_{i=1}^{N}\nabla \ell(\mathbf{w}, \mathbf{x}^{(i)}, t^{(i)}) \approx \sum_{i \in \mathcal{S}} \frac{1}{|\mathcal{S}|}\nabla \ell(\mathbf{w}, \mathbf{x}^{(i)}, t^{(i)})$$
Dealing with Big Data

$$\min_{\mathbf{w}} \frac{1}{N} \sum_{i=1}^{N} \ell(\mathbf{w}, \mathbf{x}^{(i)}, t^{(i)}) + \mathcal{R}(\mathbf{w})$$

• We need to compute at each iteration

$$\mathbf{w}_{n+1} = \mathbf{w}_n - \gamma_n \nabla L(\mathbf{w}_n)$$

with

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• This is called stochastic gradient descent

R. Urtasun (UofT)

Stochastic Gradient Descent with Momentum

• Stochastic Gradient Descent update

$$\mathbf{w}_{n+1} = \mathbf{w}_n - \gamma_n \nabla L(\mathbf{w}_n)$$

with

$$abla \mathcal{L}(\mathbf{w}_n) = \sum_{i \in \mathcal{S}} \frac{1}{|\mathcal{S}|} \nabla \ell(\mathbf{w}, \mathbf{x}^{(i)}, t^{(i)}) + \nabla \mathcal{R}(\mathbf{w})$$

Stochastic Gradient Descent with Momentum

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abla \ell(\mathbf{w}, \mathbf{x}^{(i)}, t^{(i)}) +
abla \mathcal{R}(\mathbf{w})$$

• We can also use momentum

$$\mathbf{w} \leftarrow \mathbf{w} - \gamma \Delta$$
$$\Delta \leftarrow \kappa \Delta + \nabla L$$

Stochastic Gradient Descent with Momentum

• Stochastic Gradient Descent update

$$\mathbf{w}_{n+1} = \mathbf{w}_n - \gamma_n \nabla L(\mathbf{w}_n)$$

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• We can also use momentum

$$\mathbf{w} \leftarrow \mathbf{w} - \gamma \Delta$$
$$\Delta \leftarrow \kappa \Delta + \nabla L$$

Many other variants exist

• Images can have millions of pixels, i.e., x is very high dimensional

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- Prohibitive to have fully-connected layer

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- Images can have millions of pixels, i.e., x is very high dimensional
- Prohibitive to have fully-connected layer
- We can use a locally connected layer
- This is good when the input is registered



Deep Structured Models

Locally Connected Layer



Deep Structured Models

Convolutional Neural Net

- Idea: statistics are similar at different locations (Lecun 1998)
- Connect each hidden unit to a small input patch and share the weight across space
- This is called a **convolution layer** and the network is a **convolutional network**





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$$h_j^n = \max(0, \sum_{k=1}^{K} h_k^{n-1} * w_{jk}^n)$$

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$$h_j^n = \max(0, \sum_{k=1}^K h_k^{n-1} * w_{jk}^n)$$

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$$h_j^n = \max(0, \sum_{k=1}^K h_k^{n-1} * w_{jk}^n)$$

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Pooling Layer

By "pooling" (e.g., taking max) filter responses at different locations we gain robustness to the exact spatial location of features.

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- Max Pooling: return the maximal argument
- Average Pooling: return the average of the arguments
- Other types of pooling exist: *L*₂ pooling

Pooling Layer: Receptive Field Size



If convolutional filters have size KxK and stride 1, and pooling layer has pools of size PxP, then each unit in the pooling layer depends upon a patch (at the input of the preceding conv. layer) of size:



Now let's make this very deep

Image: A matrix

• Remember from your image processing / computer vision course about filtering?

Input "image"

Filter





• If our filter was [-1,1], we got a vertical edge detector

Input "image"

Filter





Output map



• Now imagine we want to have many filters (e.g., vertical, horizontal, corners, one for dots). We will use a **filterbank**.



• So applying a filterbank to an image yields a cube-like output, a 3D matrix in which each slice is an output of convolution with one filter.



In this example our network will always expect a 224x224x3 image.

• So applying a filterbank to an image yields a cube-like output, a 3D matrix in which each slice is an output of convolution with one filter.



• Do some additional tricks. A popular one is called **max pooling**. Any idea why you would do this?



• Do some additional tricks. A popular one is called **max pooling**. Any idea why you would do this? To get **invariance to small shifts in position**.



• Now add another "layer" of filters. For each filter again do convolution, but this time with the output cube of the previous layer.



Add one more layer of filters

These filters are convolved with the output of the previous layer. The results of each convolution is again a slice in the cube on the right.

What is the dimension of each of these filters?

• Keep adding a few layers. Any idea what's the purpose of more layers? Why can't we just have a full bunch of filters in one layer?



Do it recursively

Have multiple ``layers''

[Slide Credit: Sanja Fidler, Pic adopted from: A. Krizhevsky]

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• In the end add one or two **fully** (or **densely**) connected layers. In this layer, we don't do convolution we just do a dot-product between the "filter" and the output of the previous layer.



 Add one final layer: a classification layer. Each dimension of this vector tells us the probability of the input image being of a certain class.



Add a classification ``layer".

For an input image, the value in a particular dimension of this vector tells you the probability of the corresponding object class.

• The trick is to not hand-fix the weights, but to **train** them. Train them such that when the network sees a picture of a dog, the last layer will say "dog".



• Or when the network sees a picture of a cat, the last layer will say "cat".



[Slide Credit: Sanja Fidler, Pic adopted from: A. Krizhevsky]

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• Or when the network sees a picture of a boat, the last layer will say "boat"... The more pictures the network sees, the better.



Train on **lots** of examples. Millions. Tens of millions. Wait a week for training to finish. Share your network (the weights) with others who are not fortunate enough with GPU power.

[Slide Credit: Sanja Fidler, Pic adopted from: A. Krizhevsky]

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Classification

• Once trained we feed in an image or a crop, run through the network, and read out the class with the highest probability in the last (classif) layer.



What's the class of this object?



[Slide Credit: Sanja Fidler]

Classification Performance

- Imagenet, main challenge for object classification: http://image-net.org/
- 1000 classes, 1.2M training images, 150K for test



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Architecture for Classification



Architecture for Classification



The 2012 Computer Vision Crisis



Error (5 predictions)







 What vision people like to do is take the already trained network (avoid one week of training), and remove the last classification layer. Then take the top remaining layer (the 4096 dimensional vector here) and use it as a descriptor (feature vector).



it as a descriptor. A much better descriptor than SIFT, etc.

Train your own classifier on top for your choice of classes.

[Slide Credit: Sanja Fidler]

- What vision people like to do is take the already trained network, and remove the last classification layer. Then take the top remaining layer (the 4096 dimensional vector here) and use it as a descriptor (feature vector).
- Now train your own classifier on top of these features for arbitrary classes.



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- This is quite hacky, but works miraculously well.



- What vision people like to do is take the already trained network, and remove the last classification layer. Then take the top remaining layer (the 4096 dimensional vector here) and use it as a descriptor (feature vector).
- Now train your own classifier on top of these features for arbitrary classes.
- This is quite hacky, but works miraculously well.
- Everywhere where we were using SIFT (or anything else), you can use NNs.



Caltech Results



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Caltech Results



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And Detection?

• For classification we feed in the full image to the network. But how can we perform detection?



Find all objects of interest in this image!

The Era Post-Alex Net: PASCAL VOC detection

R-CNN: Regions with CNN features



- Extract object proposals with bottom up grouping
- and then classify them using your big net

Detection Performance

• PASCAL VOC challenge: http://pascallin.ecs.soton.ac.uk/challenges/VOC/.



Figure : PASCAL has 20 object classes, 10K images for training, 10K for test

Detection Performance a Year Ago: 40.4%

A year ago, no networks:

• Results on the main recognition benchmark, the **PASCAL VOC challenge**.

	mean	aero plane	bicycle	bird	boat	bottle	bus	car	cat	chair	cow	dining table	dog	horse	motor bike	person	potted plant	sheep	sofa	train	tv/ monitor	submission date
	-	\bigtriangledown	∇	\bigtriangledown																		
segDPM ^[7]	40.4	61.4	53.4	25.6	25.2	35.5	51.7	50.6	50.8	19.3	33.8	26.8	40.4	48.3	54.4	47.1	14.8	38.7	35.0	52.8	43.1	24-Feb-2014
Boosted HOG-LBP and multi-context (LC, EGC, HLC) [7]	36.8	53.3	55.3	19.2	21.0	30.0	54.5	46.7	41.2	20.0	31.5	20.8	30.3	48.6	55.3	46.5	10.2	34.4	26.6	50.3	40.3	29-Aug-2010
MITUCLA_Hierarchy [7]	36.0	54.3	48.5	15.7	19.2	29.2	55.6	43.5	41.7	16.9	28.5	26.7	30.9	48.3	55.0	41.7	9.7	35.8	30.8	47.2	40.8	30-Aug-2010
HOGLBP_context_classification_rescore_v2 [7]	34.2	49.1	52.4	17.8	12.0	30.6	\$3.5	32.8	37.3	17.7	30.6	27.7	29.5	51.9	56.3	44.2	9.6	14.8	27.9	49.5	38.4	30-Aug-2010
LSVM-MDPM [?]	33.7	52.4	54.3	13.0	15.6	35.1	54.2	49.1	31.8	15.5	26.2	13.5	21.5	45.4	51.6	47.5	9.1	35.1	19.4	46.6	38.0	26-Aug-2010
UOCTTI_LSVM_MDPM [?]	33.4	49.2	53.8	13.1	15.3	35.5	53.4	49.7	27.0	17.2	28.8	14.7	17.8	46.4	51.2	47.7	10.8	34.2	20.7	43.8	38.3	21-May-2012
Detection Monkey [7]	32.9	56.7	39.8	16.8	12.2	13.8	44.9	36.9	47.7	12.1	26.9	26.5	37.2	42.1	51.9	25.7	12.1	37.8	33.0	41.5	41.7	30-Aug-2010
RM^2C [7]	32.8	49.8	50.6	15.1	15.5	28.5	51.1	42.2	30.5	17.3	28.3	12.4	26.0	45.6	51.8	41.4	12.6	30.4	26.1	44.0	37.6	29-Oct-2013
UOCTTI_LSVM_MDPM ^[7]	32.2	48.2	52.2	14.8	13.8	28.7	\$3.2	44.9	26.0	18.4	24.4	13.7	23.1	45.8	50.5	43.7	9.8	31.1	21.5	44.4	35.7	11-May-2012
GroupLoc [7]	31.9	58.4	39.6	18.0	13.3	11.1	46.4	37.8	43.9	10.3	27.5	20.8	36.0	39.4	48.5	22.9	13.0	36.9	30.5	41.2	41.9	30-Aug-2010
UOCTTI_LSVM_MDPM ^[7]	29.6	45.6	49.0	11.0	11.6	27.2	50.5	43.1	23.6	17.2	23.2	10.7	20.5	42.5	44.5	41.3	8.7	29.0	18.7	40.0	34.5	21-May-2012
Bonn_FGT_Segm [7]	26.1	52.7	33.7	13.2	11.0	14.2	43.2	31.9	35.6	5.8	25.4	14.4	20.6	38.1	41.7	25.0	5.8	26.3	18.1	37.6	28.1	30-Aug-2010
HOG-LBP + DHOG bag of words, SVM [7]	23.5	40.4	34.7	2.7	8.4	26.0	43.1	33.8	17.2	11.2	14.3	14.5	14.9	31.8	37.3	30.0	6.4	25.2	11.6	30.0	35.7	30-Aug-2010
Svr-Segm [7]	23.4	50.5	24.5	17.1	13.3	10.9	39.5	32.9	36.5	5.6	16.0	6.6	22.3	24.9	29.0	29.8	6.7	28.4	13.3	32.1	27.2	30-Aug-2010
HOG-LBP Linear SVM ^[7]	22.1	37.9	33.7	2.7	6.5	25.3	37.5	33.1	15.5	10.9	12.3	12.5	13.7	29.7	34.5	33.8	7.2	22.9	9.9	28.9	34.1	29-Aug-2010
HOG+LBP+LTP+PLS2ROOTS [7]	17.5	32.7	29.7	0.8	1.1	19.9	39.4	27.5	8.6	4.5	8.1	6.3	11.0	22.9	34.1	24.6	3.1	24.0	2.0	23.5	27.0	31-Aug-2010
RandomParts [7]	14.2	23.8	31.7	1.2	3.4	11.1	29.7	19.5	14.2	0.8	11.1	7.0	4.7	16.4	31.5	16.0	1.1	15.6	10.2	14.7	21.0	25-Aug-2010
SIFT-GMM-MKL2 [7]	8.3	20.0	14.5	3.8	1.2	0.5	17.6	8.1	28.5	0.1	2.9	3.1	17.5	7.2	18.8	3.3	0.8	2.9	6.3	7.6	1.1	30-Aug-2010
UC3M_Generative_Discriminative [7]	6.3	15.8	5.5	5.6	2.3	0.3	10.2	5.4	12.6	0.5	5.6	4.5	7.7	11.3	12.6	5.3	1.5	2.0	5.9	9.1	3.2	30-Aug-2010
SIFT-GMM-MKL [7]	2.3	10.6	1.6	1.2	0.9	0.1	2.8	1.6	6.7	0.1	2.0	0.4	3.0	2.0	4.4	2.0	0.3	1.1	1.2	2.1	1.9	30-Aug-2010

Figure : Leading method segDPM (ours). Those were the good times...

S. Fidler, R. Mottaghi, A. Yuille, R. Urtasun, Bottom-up Segmentation for Top-down Detection, CVPR'13

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The Era Post-Alex Net: PASCAL VOC detection

\triangleright	R-CNN [?]	50.2
\triangleright	BERKELEY POSELETS [?]	-
\triangleright	poselets [?]	-
\triangleright	** UCI_LSVM-MDPM-10X ** [?]	-
\triangleright	Head-Detect-Segment [?]	-

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- So networks turn out to be great.
- Everything is deep, even if it's shallow!
- Companies leading the competitions: ImageNet, KITTI, but not yet PASCAL
- At this point Google, Facebook, Microsoft, Baidu "steal" most neural network professors from academia.
- · · · and a lot of our good students :(

So Neural Networks are Great

• But to train the networks you need quite a bit of computational power. So what do you do?



So Neural Networks are Great

• Buy even more.



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Deep Structured Models

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So Neural Networks are Great

• And train more layers. 16 instead of 7 before. 144 million parameters.

add more layers



Figure : K. Simonyan, A. Zisserman, Very Deep Convolutional Networks for Large-Scale Image Recognition. arXiv 2014

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Deep Structured Models

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The Era Post-Alex Net: PASCAL VOC detection

۲	Fast R-CNN + YOLO [?]	70.8
\triangleright	Fast R-CNN VGG16 extra data [?]	68.8
\triangleright	segDeepM [?]	67.2
\triangleright	BabyLearning [?]	63.8
\triangleright	R-CNN (bbox reg) [?]	62.9
\triangleright	R-CNN [?]	59.8
\triangleright	Feature Edit [?]	56.4
\triangleright	YOLO [?]	55.3
\triangleright	R-CNN (bbox reg) [?]	53.7
\triangleright	R-CNN [?]	50.2
\triangleright	poselets ^[?]	-
\triangleright	Head-Detect-Segment [?]	-
\triangleright	BERKELEY POSELETS [?]	-
\triangleright	** UCI_LSVM-MDPM-10X ** [?]	-

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- More to come in Part II

Practical Tips

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 - 8 Random [Bergstra & Bengio JMLR 2012]
 - Bayesian optimization [Whetlab Toronto]

Good to Know

• ALWAYS check gradients numerically by finite differences!

Image: A matrix

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Figure : from M. Ranzato

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- Visualize features (feature maps need to be uncorrelated) and have high variance.
- Visualize parameters



Figure : from M. Ranzato

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Image: A matrix

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 - Decrease learning rate

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 - GPU, distrib. framework, make net smaller

[Slide credit: M. Ranzato]

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- Sparsity in the hidden units
- Multi-task learning

[Slide credit: M. Ranzato]

Software

- Torch7: learning library that supports neural net training http://www.torch.ch http://code.cogbits.com/wiki/doku.php (tutorial with demos by C. Farabet) https://github.com/sermanet/OverFeat
- Python-based learning library (U. Montreal) http://deeplearning.net/software/theano/ (does automatic differentiation
- Efficient CUDA kernels for ConvNets (Krizhevsky) code.google.com/p/cuda-convnet
- Caffe (Yangqing Jia) http://caffe.berkeleyvision.org
- Deep Structured Models http://www.alexander-schwing.de/ (soon available)

[Slide Credit: M. Ranzato]

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Part II: Deep Structured Learning

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- Theoretical Understanding
- Onsupervised Learning
- Structured models

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• Many Vision Problems are complex and involve predicting many random variables that are statistically related



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 Complex mapping F(x, y, w) to predict output y given input x through a series of matrix multiplications, non-linearities and pooling operations



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- We typically train the network to predict one random variable (e.g., ImageNet) by minimizing cross-entropy
- Multi-task extensions: sum the loss of each task, and share part of the features (e.g., segmentation)
- Use an MRF as a post processing step

<u>PROBLEM</u>: How can we take into account complex dependencies when predicting multiple variables?

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SOLUTION: Graphical models

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Graphical Models

• Convenient tool to illustrate dependencies among random variables



• Widespread usage among different fields: vision, NLP, comp. bio, · · ·

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Compact Notation

• In Computer Vision we usually express



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• For the purpose of this talk we are going to use a more compact notation

$$\mathsf{E}(\mathbf{y},\mathbf{w}) = -\sum_{r\in\mathcal{R}} f_r(\mathbf{y}_r,\mathbf{w})$$

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- r is a region and R is the set of all regions
- y_r is of any order
- The functions f_r are a function of parameters **w**

Continuous vs Discrete MRFs

$$E(\mathbf{y},\mathbf{w}) = -\sum_{r\in\mathcal{R}} f_r(\mathbf{y}_r,\mathbf{w})$$

• Discrete MRFs: $y_i \in \{1, \cdots, C_i\}$



• Continuous MRFs: $y_i \in \mathcal{Y} \subseteq \mathbb{R}$



• Hybrid MRFs with continuous and discrete variables

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- Hybrid MRFs with continuous and discrete variables
- Today's talk: only discrete MRFs

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Deep Structured Models

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Probabilistic Interpretation

• The energy is defined as

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$$p(\mathbf{y}; \mathbf{w}) = \frac{1}{Z} \exp\left(\sum_{r \in \mathcal{R}} f_r(\mathbf{y}_r, \mathbf{w})\right)$$

with $Z(\mathbf{w}) = \sum_{\mathbf{y}} \exp\left(\sum_{r \in \mathcal{R}} f_r(\mathbf{y}_r, \mathbf{w})\right)$ the partition function

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$$p(\mathbf{y}|\mathbf{x};\mathbf{w}) = \frac{1}{Z(\mathbf{x})} \exp\left(\sum_{r \in \mathcal{R}} f_r(\mathbf{x},\mathbf{y}_r,\mathbf{w})\right)$$

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Probabilistic Inference: We might want to compute p(y_r) for any possible subset of variables r, or p(y_r|y_p) for any subset r and p

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Very difficult tasks in general (i.e., NP-hard). Some exceptions, e.g., low-tree width models and binary MRFs with sub-modular energies

Learning in CRFs

- Given a training set of N pairs $(\mathbf{x}, \mathbf{y}) \in \mathcal{D}$, we want to estimate the functions $f_r(\mathbf{x}, \mathbf{y}_r, \mathbf{w})$
- As these functions are parametric, this is equivalent to estimating w

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- We would like to do this by minimizing the empirical loss

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• Very difficult, instead we minimize the sum of a surrogate (typically convex) loss and a regularizer

$$\min_{\mathbf{w}} R(\mathbf{w}) + \frac{C}{N} \sum_{(\mathbf{x}, \mathbf{y}) \in \mathcal{D}} \bar{\ell}(\mathbf{x}, \mathbf{y}, \mathbf{w})$$

More on Learning in CRFs

- Given a training set of N pairs $(\mathbf{x}, \mathbf{y}) \in D$, we want to estimate the functions $f_r(\mathbf{y}, \mathbf{x}, \mathbf{w})$
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• The surrogate loss $\overline{\ell}$: hinge-loss, log-loss

$$\begin{split} \bar{\ell}_{log}(\mathbf{x},\mathbf{y},\mathbf{w}) &= -\ln p_{\mathbf{x},\mathbf{y}}(\mathbf{y};\mathbf{w}).\\ \bar{\ell}_{hinge}(\mathbf{x},\mathbf{y},\mathbf{w}) &= \max_{\hat{\mathbf{y}}\in\mathcal{Y}} \left\{ \ell(\mathbf{y},\hat{\mathbf{y}}) - \mathbf{w}^{\top} \Phi(\mathbf{x},\hat{\mathbf{y}}) + \mathbf{w}^{\top} \Phi(\mathbf{x},\mathbf{y}) \right\} \end{split}$$

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• The assumption is that the model is log-linear

$$E(\mathbf{x},\mathbf{y},\mathbf{w}) = -F(\mathbf{x},\mathbf{y},\mathbf{w}) = -\mathbf{w}^{T}\phi(\mathbf{x},\mathbf{y})$$

and the features decompose in a graph

$$\mathbf{w}^T \phi(\mathbf{x}, \mathbf{y}) = \sum_{r \in \mathcal{R}} \mathbf{w}_r^T \phi(\mathbf{x}, \mathbf{y})$$

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PROBLEM: How can we remove the log-linear restriction?

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Image: Image:

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<u>PROBLEM</u>: How can we remove the log-linear restriction?

SOLUTION: Deep Structured Models

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Standard CNN



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Standard CNN



• Deep Structured Models



Learning

Probability of a configuration **y**:

$$p(\mathbf{y} \mid \mathbf{x}; \mathbf{w}) = \frac{1}{Z(\mathbf{x}, \mathbf{w})} \exp F(\mathbf{x}, \mathbf{y}, \mathbf{w})$$
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Maximize the likelihood of training data via

$$\begin{split} \mathbf{w}^* &= \arg \max_{\mathbf{w}} \prod_{(\mathbf{x}, \mathbf{y}) \in \mathcal{D}} p(\mathbf{y} | \mathbf{x}; \mathbf{w}) \\ &= \arg \max_{\mathbf{w}} \sum_{(\mathbf{x}, \mathbf{y}) \in \mathcal{D}} \left(F(\mathbf{x}, \mathbf{y}, \mathbf{w}) - \ln \sum_{\hat{\mathbf{y}} \in \mathcal{Y}} \exp F(\mathbf{x}, \mathbf{y}, \mathbf{w}) \right) \end{split}$$

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Maximum likelihood is equivalent to maximizing cross-entropy when the target distribution $p_{(\mathbf{x},\mathbf{y}),\mathrm{tg}}(\hat{\mathbf{y}}) = \delta(\hat{\mathbf{y}} = \mathbf{y})$

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Gradient Ascent on Cross Entropy

Program of interest:

$$\max_{\mathbf{w}} \sum_{(\mathbf{x}, \mathbf{y}) \in \mathcal{D}, \hat{\mathbf{y}}} p_{(\mathbf{x}, \mathbf{y}), \mathrm{tg}}(\hat{\mathbf{y}}) \ln p(\hat{\mathbf{y}} \mid \mathbf{x}; \mathbf{w})$$

Optimize via gradient ascent

$$\frac{\partial}{\partial \mathbf{w}} \sum_{(\mathbf{x}, \mathbf{y}) \in \mathcal{D}, \hat{\mathbf{y}}} p_{(\mathbf{x}, \mathbf{y}), tg}(\hat{\mathbf{y}}) \ln p(\hat{\mathbf{y}} \mid \mathbf{x}; \mathbf{w})$$

$$= \sum_{(\mathbf{x}, \mathbf{y}) \in \mathcal{D}, \hat{\mathbf{y}}} \left(p_{(\mathbf{x}, \mathbf{y}), tg}(\hat{\mathbf{y}}) - p(\hat{\mathbf{y}} \mid \mathbf{x}; \mathbf{w}) \right) \frac{\partial}{\partial \mathbf{w}} F(\hat{\mathbf{y}}, \mathbf{x}, \mathbf{w})$$

$$= \sum_{(\mathbf{x}, \mathbf{y}) \in \mathcal{D}} \left(\mathbb{E}_{p_{(\mathbf{x}, \mathbf{y}), tg}} \left[\frac{\partial}{\partial w} F(\hat{\mathbf{y}}, \mathbf{x}, \mathbf{w}) \right] - \mathbb{E}_{p_{(\mathbf{x}, \mathbf{y})}} \left[\frac{\partial}{\partial \mathbf{w}} F(\hat{\mathbf{y}}, \mathbf{x}, \mathbf{w}) \right] \right)$$
moment matching

- Compute predicted distribution $p(\hat{\mathbf{y}} \mid \mathbf{x}; \mathbf{w})$
- Use chain rule to pass back difference between prediction and observation

R. Urtasun (UofT)

[Peng et al. NIPS'09]

Repeat until stopping criteria

- **1** Forward pass to compute $F(\mathbf{y}, \mathbf{x}, \mathbf{w})$
- **2** Compute $p(\mathbf{y} \mid \mathbf{x}, \mathbf{w})$
- 3 Backward pass via chain rule to obtain gradient
- Update parameters w

[Peng et al. NIPS'09]

Repeat until stopping criteria

- **1** Forward pass to compute $F(\mathbf{y}, \mathbf{x}, \mathbf{w})$
- **2** Compute $p(\mathbf{y} \mid \mathbf{x}, \mathbf{w})$
- Sackward pass via chain rule to obtain gradient
- Update parameters w

What is the PROBLEM?

[Peng et al. NIPS'09]

Repeat until stopping criteria

- **(**) Forward pass to compute $F(\mathbf{y}, \mathbf{x}, \mathbf{w})$
- **2** Compute $p(\mathbf{y} \mid \mathbf{x}, \mathbf{w})$

Sackward pass via chain rule to obtain gradient

Opdate parameters w

What is the PROBLEM?

- How do we even represent $F(\mathbf{y}, \mathbf{x}, \mathbf{w})$ if \mathcal{Y} is large?
- How do we compute $p(\mathbf{y} \mid \mathbf{x}, \mathbf{w})$?

() Use the graphical model $F(\mathbf{y}, \mathbf{x}, \mathbf{w}) = \sum_{r} f_r(\mathbf{y}_r, \mathbf{x}, \mathbf{w})$

$$\frac{\partial}{\partial \mathbf{w}} \sum_{(\mathbf{x}, \mathbf{y}) \in \mathcal{D}, \hat{\mathbf{y}}} p_{(\mathbf{x}, \mathbf{y}), \mathrm{tg}}(\hat{\mathbf{y}}) \ln p(\hat{\mathbf{y}} \mid \mathbf{x}; \mathbf{w})$$

$$= \sum_{(\mathbf{x}, \mathbf{y}) \in \mathcal{D}, \mathbf{r}} \left(\mathbb{E}_{p_{(\mathbf{x}, \mathbf{y}), \mathbf{r}, \mathrm{tg}}} \left[\frac{\partial}{\partial \mathbf{w}} f_{\mathbf{r}}(\hat{\mathbf{y}}_{\mathbf{r}}, \mathbf{x}, \mathbf{w}) \right] - \mathbb{E}_{p_{(\mathbf{x}, \mathbf{y}), \mathbf{r}}} \left[\frac{\partial}{\partial \mathbf{w}} f_{\mathbf{r}}(\hat{\mathbf{y}}_{\mathbf{r}}, \mathbf{x}, \mathbf{w}) \right] \right)$$

() Use the graphical model $F(\mathbf{y}, \mathbf{x}, \mathbf{w}) = \sum_{r} f_r(\mathbf{y}_r, \mathbf{x}, \mathbf{w})$

$$\begin{aligned} \frac{\partial}{\partial \mathbf{w}} & \sum_{(\mathbf{x}, \mathbf{y}) \in \mathcal{D}, \hat{\mathbf{y}}} p_{(\mathbf{x}, \mathbf{y}), \mathrm{tg}}(\hat{\mathbf{y}}) \ln p(\hat{\mathbf{y}} \mid \mathbf{x}; \mathbf{w}) \\ &= \sum_{(\mathbf{x}, \mathbf{y}) \in \mathcal{D}, r} \left(\mathbb{E}_{p_{(\mathbf{x}, \mathbf{y}), r, \mathrm{tg}}} \left[\frac{\partial}{\partial \mathbf{w}} f_r(\hat{\mathbf{y}}_r, \mathbf{x}, \mathbf{w}) \right] - \mathbb{E}_{p_{(\mathbf{x}, \mathbf{y}), r}} \left[\frac{\partial}{\partial \mathbf{w}} f_r(\hat{\mathbf{y}}_r, \mathbf{x}, \mathbf{w}) \right] \right) \end{aligned}$$

2 Approximate marginals $p_r(\hat{\mathbf{y}}_r | \mathbf{x}, \mathbf{w})$ via beliefs $b_r(\hat{\mathbf{y}}_r | \mathbf{x}, \mathbf{w})$ computed by:

- Sampling methods
- Variational methods

Deep Structured Learning (algo 2)

[Schwing & Urtasun Arxiv'15, Zheng et al. Arxiv'15]

Repeat until stopping criteria

1 Forward pass to compute the $f_r(\mathbf{y}_r, \mathbf{x}, \mathbf{w})$

- Compute the b_r(y_r | x, w) by running approximated inference
- Backward pass via chain rule to obtain gradient

Update parameters w

Deep Structured Learning (algo 2)

[Schwing & Urtasun Arxiv'15, Zheng et al. Arxiv'15]



<u>PROBLEM</u>: We have to run inference in the graphical model every time we want to update the weights

Dealing with large number $|\mathcal{D}|$ of training examples:

- Parallelized across samples (any number of machines and GPUs)
- Usage of mini batches

Dealing with large number $|\mathcal{D}|$ of training examples:

- Parallelized across samples (any number of machines and GPUs)
- Usage of mini batches

Dealing with large output spaces $\mathcal{Y}:$

- Variational approximations
- Blending of learning and inference

Approximated Deep Structured Learning

[Schwing & Urtasun Arxiv'15]

Sample parallel implementation:

Partition data \mathcal{D} onto compute nodes Repeat until stopping criteria	
1	Each compute node uses GPU for CNN Forward pass to compute $f_r(\mathbf{y}_r, \mathbf{x}, \mathbf{w})$
2	Each compute node estimates beliefs $b_r(\mathbf{y}_r \mid \mathbf{x}, \mathbf{w})$ for assigned samples
3	Backpropagation of difference using GPU to obtain machine local gradient
4	Synchronize gradient across all machines using MPI
5	Update parameters w

Better Option: Interleaving Learning and Inference

• Use LP relaxation instead

$$\min_{\mathbf{w}} \sum_{(\mathbf{x},\mathbf{y})\in\mathcal{D}} \left(\max_{b_{(x,y)}\in\mathcal{C}_{(x,y)}} \left\{ \sum_{r,\hat{\mathbf{y}}_r} b_{(x,y),r}(\hat{\mathbf{y}}_r) f_r(\mathbf{x},\hat{\mathbf{y}}_r;\mathbf{w}) + \sum_r \epsilon c_r H(b_{(x,y),r}) \right\} - F(\mathbf{x},\mathbf{y};\mathbf{w}) \right)$$
Better Option: Interleaving Learning and Inference

• Use LP relaxation instead

$$\min_{\mathbf{w}} \sum_{(\mathbf{x}, \mathbf{y}) \in \mathcal{D}} \left(\max_{b_{(x, y)} \in \mathcal{C}_{(x, y)}} \left\{ \sum_{r, \hat{\mathbf{y}}_r} b_{(x, y), r}(\hat{\mathbf{y}}_r) f_r(\mathbf{x}, \hat{\mathbf{y}}_r; \mathbf{w}) + \sum_r \epsilon c_r H(b_{(x, y), r}) \right\} - F(\mathbf{x}, \mathbf{y}; \mathbf{w}) \right)$$

• More efficient algorithm by blending min. w.r.t. w and max. of the beliefs b

Better Option: Interleaving Learning and Inference

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- More efficient algorithm by blending min. w.r.t. ${\bf w}$ and max. of the beliefs b
- After introducing Lagrange multipliers λ , the dual becomes

$$\min_{\mathbf{w},\lambda} \sum_{(\mathbf{x},\mathbf{y}),r} \epsilon c_r \ln \sum_{\hat{\mathbf{y}}_r} \exp \frac{f_r(\mathbf{x}, \hat{\mathbf{y}}_r; \mathbf{w}) + \sum_{c \in C(r)} \lambda_{(x,y),c \to r}(\hat{\mathbf{y}}_c) - \sum_{p \in P(r)} \lambda_{(x,y),r \to p}(\hat{\mathbf{y}}_r)}{\epsilon c_r} - \overline{F}(\mathbf{w}).$$

with $\overline{F}(\mathbf{w}) = \sum_{(\mathbf{x},\mathbf{y})\in\mathcal{D}} F(\mathbf{x},\mathbf{y};\mathbf{w})$ the sum of empirical function observations

Better Option: Interleaving Learning and Inference

• Use LP relaxation instead

$$\min_{\mathbf{w}} \sum_{(\mathbf{x},\mathbf{y})\in\mathcal{D}} \left(\max_{b_{(x,y)}\in\mathcal{C}_{(x,y)}} \left\{ \sum_{r,\hat{\mathbf{y}}_r} b_{(x,y),r}(\hat{\mathbf{y}}_r) f_r(\mathbf{x},\hat{\mathbf{y}}_r;\mathbf{w}) + \sum_r \epsilon c_r H(b_{(x,y),r}) \right\} - F(\mathbf{x},\mathbf{y};\mathbf{w}) \right)$$

- More efficient algorithm by blending min. w.r.t. ${\bf w}$ and max. of the beliefs b
- After introducing Lagrange multipliers λ , the dual becomes

$$\min_{\mathbf{w},\lambda} \sum_{(\mathbf{x},\mathbf{y}),r} \epsilon c_r \ln \sum_{\hat{\mathbf{y}}_r} \exp \frac{f_r(\mathbf{x}, \hat{\mathbf{y}}_r; \mathbf{w}) + \sum_{c \in C(r)} \lambda_{(x,y),c \to r}(\hat{\mathbf{y}}_c) - \sum_{p \in P(r)} \lambda_{(x,y),r \to p}(\hat{\mathbf{y}}_r)}{\epsilon c_r} - \overline{F}(\mathbf{w}).$$

with $\overline{F}(\mathbf{w}) = \sum_{(\mathbf{x},\mathbf{y})\in\mathcal{D}} F(\mathbf{x},\mathbf{y};\mathbf{w})$ the sum of empirical function observations

• We can then do block coordinate descent to solve the minimization problem, and we get the following algorithm ···

[Chen & Schwing & Yuille & Urtasun ICML'15]



Deep Structured Learning (algo 4)

[Chen & Schwing & Yuille & Urtasun ICML'15]

Sample parallel implementation:

Partition data ${\cal D}$ onto compute nodes Repeat until stopping criteria
Each compute node uses GPU for CNN Forward pass to compute f _r (y _r , x, w)
2 Each compute node updates (some) messages λ
Backpropagation of difference using GPU to obtain machine local gradient
Synchronize gradient across all machines using MPI
Opdate parameters w



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Application 1: Character Recognition

- Task: Word Recognition from a fixed vocabulary of 50 words, 28 × 28 sized image patches
- Characters have complex backgrounds and suffer many different distortions
- Training, validation and test set sizes are 10k, 2k and 2k variations of words



Results

- Graphical model has 5 nodes, MLP for each unary and non-parametric pairwise potentials
- Joint training, structured, deep and more capacity helps

Grap	MLP	Method	$H_1 = 128$	$H_1 = 256$	$H_1 = 512$	$H_1 = 768$	$H_1 = 1024$
		Unary only	8.60 / 61.32	10.80 / 64.41	12.50 / 65.69	12.95 / 66.66	13.40 / 67.02
1ct	1104	JointTrain	16.80 / 65.28	25.20 / 70.75	31.80 / 74.90	33.05 / 76.42	34.30 / 77.02
150	Ilay	PwTrain	12.70 / 64.35	18.00 / 68.27	22.80 / 71.29	23.25 / 72.62	26.30 / 73.96
		PreTrainJoint	20.65 / 67.42	25.70 / 71.65	31.70 / 75.56	34.50 / 77.14	35.85 / 78.05
		JointTrain	25.50 / 67.13	34.60 / 73.19	45.55 / 79.60	51.55 / 82.37	54.05 / 83.57
2nd	1lay	PwTrain	10.05 / 58.90	14.10 / 63.44	18.10 / 67.31	20.40 / 70.14	22.20 / 71.25
		PreTrainJoint	28.15 / 69.07	36.85 / 75.21	45.75 / 80.09	50.10 / 82.30	52.25 / 83.39
		$H_1 = 512$	$H_2 = 32$	$H_2 = 64$	$H_2 = 128$	$H_2 = 256$	$H_2 = 512$
		$H_1 = 512$ Unary only	$H_2 = 32$ 15.25 / 69.04	$H_2 = 64$ 18.15 / 70.66	$H_2 = 128$ 19.00 / 71.43	$H_2 = 256$ 19.20 / 72.06	$\frac{H_2 = 512}{20.40 / 72.51}$
1ct	2121	$H_1 = 512$ Unary only JointTrain	$H_2 = 32$ 15.25 / 69.04 35.95 / 76.92	$H_2 = 64 \\ 18.15 / 70.66 \\ 43.80 / 81.64$	$H_2 = 128$ $19.00 / 71.43$ $44.75 / 82.22$	$H_2 = 256$ $19.20 / 72.06$ $46.00 / 82.96$	$H_2 = 512$ 20.40 / 72.51 47.70 / 83.64
1st	2lay	$H_1 = 512$ Unary only JointTrain PwTrain	$H_2 = 32 \\ 15.25 / 69.04 \\ 35.95 / 76.92 \\ 34.85 / 79.11$	$H_2 = 64 \\ 18.15 / 70.66 \\ 43.80 / 81.64 \\ 38.95 / 80.93$	$H_2 = 128 \\ 19.00 / 71.43 \\ 44.75 / 82.22 \\ 42.75 / 82.38$	$H_2 = 256 \\ 19.20 / 72.06 \\ 46.00 / 82.96 \\ 45.10 / 83.67$	$H_2 = 512$ 20.40 / 72.51 47.70 / 83.64 45.75 / 83.88
1st	2lay	$H_1 = 512$ Unary only JointTrain PwTrain PreTrainJoint	$H_2 = 32$ $15.25 / 69.04$ $35.95 / 76.92$ $34.85 / 79.11$ $42.25 / 81.10$	$H_2 = 64$ $18.15 / 70.66$ $43.80 / 81.64$ $38.95 / 80.93$ $44.85 / 82.96$	$H_2 = 128$ $19.00 / 71.43$ $44.75 / 82.22$ $42.75 / 82.38$ $46.85 / 83.50$	$H_2 = 256$ $19.20 / 72.06$ $46.00 / 82.96$ $45.10 / 83.67$ $47.95 / 84.21$	$H_2 = 512$ 20.40 / 72.51 47.70 / 83.64 45.75 / 83.88 47.05 / 84.08
1st	2lay	H ₁ = 512 Unary only JointTrain PwTrain PreTrainJoint JointTrain	$H_2 = 32$ $15.25 / 69.04$ $35.95 / 76.92$ $34.85 / 79.11$ $42.25 / 81.10$ $54.65 / 83.98$	$H_2 = 64$ $18.15 / 70.66$ $43.80 / 81.64$ $38.95 / 80.93$ $44.85 / 82.96$ $61.80 / 87.30$	$H_2 = 128$ $19.00 / 71.43$ $44.75 / 82.22$ $42.75 / 82.38$ $46.85 / 83.50$ $66.15 / 89.09$	$H_2 = 256$ $19.20 / 72.06$ $46.00 / 82.96$ $45.10 / 83.67$ $47.95 / 84.21$ $64.85 / 88.93$	$H_2 = 512$ 20.40 / 72.51 47.70 / 83.64 45.75 / 83.88 47.05 / 84.08 68.00 / 89.96
1st 2nd	2lay 2lay	H ₁ = 512 Unary only JointTrain PwTrain PreTrainJoint JointTrain PwTrain	$H_2 = 32$ $15.25 / 69.04$ $35.95 / 76.92$ $34.85 / 79.11$ $42.25 / 81.10$ $54.65 / 83.98$ $39.95 / 81.14$	$H_2 = 64$ $18.15 / 70.66$ $43.80 / 81.64$ $38.95 / 80.93$ $44.85 / 82.96$ $61.80 / 87.30$ $48.25 / 84.45$	$\begin{array}{c} H_2 = 128 \\ 19.00 \ / \ 71.43 \\ 44.75 \ / \ 82.22 \\ 42.75 \ / \ 82.38 \\ \hline \textbf{46.85} \ / \ \textbf{83.50} \\ \hline \textbf{66.15} \ / \ \textbf{89.09} \\ 52.65 \ / \ \textbf{86.24} \end{array}$	$\begin{array}{r} H_2 = 256 \\ 19.20 \ / \ 72.06 \\ 46.00 \ / \ 82.96 \\ 45.10 \ / \ 83.67 \\ \textbf{47.95} \ / \ \textbf{84.21} \\ 64.85 \ / \ 88.93 \\ 57.10 \ / \ 87.61 \end{array}$	H ₂ = 512 20.40 / 72.51 47.70 / 83.64 45.75 / 83.88 47.05 / 84.08 68.00 / 89.96 62.90 / 89.49

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Unary weights



abcdefghijklmnopqrstuvwxyz

distance-1 edges



distance-2 edges

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Example 2: Image Tagging

[Chen & Schwing & Yuille & Urtasun ICML'15]

- Flickr dataset: 38 possible tags, $|\mathcal{Y}| = 2^{38}$
- 10k training, 10k test examples

Training method	Prediction error [%]
Unary only	9.36
Piecewise	7.70
Joint (with pre-training)	7.25



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Visual results







female/indoor/portrait female/indoor/portrait

sky/plant life/tree sky/plant life/tree

water/animals/sea water/animals/sky



animals/dog/indoor animals/dog



indoor/flower/plant life

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Only part of the correlations are shown for clarity

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Example 3: Semantic Segmentation

[Chen et al. ICLR'15; Krähenbühl & Koltun NIPS'11,ICML'13; Zhen et al. Arxiv'15; Schwing & Urtasun Arxiv'15]

- $|\mathcal{Y}| = 21^{350\cdot 500}$, pprox 10k training, pprox 1500 test examples
- Oxford-net pre trained on PASCAL, predicts $40 \times 40 + upsampling$
- The graphical model is a fully connected CRF with Gaussian potentials
- Inference using (algo2), with mean-field as approx. inference



[Chen et al. ICLR'15; Krähenbühl & Koltun NIPS'11,ICML'13; Zhen et al. Arxiv'15; Schwing & Urtasun Arxiv'15]

- $|\mathcal{Y}| = 21^{350 \cdot 500}$, $\approx 10k$ training, ≈ 1500 test examples
- Oxford-net pre trained on PASCAL, predicts $40 \times 40 + upsampling$
- The graphical model is a fully connected CRF with Gaussian potentials
- Inference using (algo2), with mean-field as approx. inference

Training method	Mean IoU [%]
Unary only	61.476
Joint	64.060

[Chen et al. ICLR'15; Krähenbühl & Koltun NIPS'11,ICML'13; Zhen et al. Arxiv'15; Schwing & Urtasun Arxiv'15]

- $|\mathcal{Y}| = 21^{350 \cdot 500}$, $\approx 10k$ training, ≈ 1500 test examples
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- Inference using (algo2), with mean-field as approx. inference

Training method	Mean IoU [%]
Unary only	61.476
Joint	64.060

• **Disclaimer**: Much better results now with a few tricks. Zheng et al. 15 is now at 74.7%!

Visual results



















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Example 4: 3D Object Proposals for Detection

• Use structured prediction to learn to propose object candidates (i.e., grouping)



(image)

(stereo)

(depth-feat)

(prior)

• Use deep learning to do final detection: OxfordNet



• Only 1.2s to generate proposals

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Figure : Proposal recall: 0.7 overlap threshold for Car, and 0.5 for rest.

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(Bottom) Cyclists.

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[X.	Chen, K.	Kundu	and S.	Fidler	and R.	Urtasun,	On Arxiv soo	n]
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	Cars			Pedestrians			Cyclists		
	Easy	Moderate	Hard	Easy	Moderate	Hard	Easy	Moderate	Hard
LSVM-MDPM-sv	68.02	56.48	44.18	47.74	39.36	35.95	35.04	27.50	26.21
SquaresICF	-	-	-	57.33	44.42	40.08	-	-	-
DPM-C8B1	74.33	60.99	47.16	38.96	29.03	25.61	43.49	29.04	26.20
MDPM-un-BB	71.19	62.16	48.43	-	-	-	-	-	-
DPM-VOC+VP	74.95	64.71	48.76	59.48	44.86	40.37	42.43	31.08	28.23
OC-DPM	74.94	65.95	53.86	-	-	-	-	-	-
AOG	84.36	71.88	59.27	-	-	-	-	-	-
SubCat	84.14	75.46	59.71	54.67	42.34	37.95	-	-	-
DA-DPM	-	-	-	56.36	45.51	41.08	-	-	-
Fusion-DPM	-	-	-	59.51	46.67	42.05	-	-	-
R-CNN	-	-	-	61.61	50.13	44.79	-	-	-
FilteredICF	-	-	-	61.14	53.98	49.29	-	-	-
pAUCEnsT	-	-	-	65.26	54.49	48.60	51.62	38.03	33.38
MV-RGBD-RF	-	-	-	70.21	54.56	51.25	54.02	39.72	34.82
3DVP	87.46	75.77	65.38	-	-	-	-	-	-
Regionlets	84.75	76.45	59.70	73.14	61.15	55.21	70.41	58.72	51.83
Ours	88.33	87.14	76.11	70.16	59.35	52.76	77.94	67.35	59.49

Table : Average Precision (AP) (in %) on the test set of the KITTI Object Detection Benchmark.

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[X. Chen, K. Kundu and S. Fidler and R. Urtasun, On Arxiv soon]

				-					
	Cars			Pedestrians			Cyclists		
	Easy	Mod.	Hard	Easy	Mod.	Hard	Easy	Mod.	Hard
AOG	43.81	38.21	31.53	-	-	-	-	-	-
DPM-C8B1	59.51	50.32	39.22	31.08	23.37	20.72	27.25	19.25	17.95
LSVM-MDPM-sv	67.27	55.77	43.59	43.58	35.49	32.42	27.54	22.07	21.45
DPM-VOC+VP	72.28	61.84	46.54	53.55	39.83	35.73 /	30.52	23.17	21.58
OC-DPM	73.50	64.42	52.40	-	-	-	-	-	-
SubCat	83.41	74.42	58.83	44.32	34.18	30.76	- 1	-	-
3DVP	86.92	74.59	64.11	-	-	-	-	-	-
Ours	83.03	80.21	69.60	48.58	40.56	36.08	57.72	48.21	42.72

Table : AOS scores on the KITTI Object Detection and Orientation Benchmark (test set).

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Car Results

[X. Chen, K. Kundu, Y. Zhu, S. Fidler and R. Urtasun, On Arxiv soon]

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Pedestrian Results

[X. Chen, K. Kundu, Y. Zhu, S. Fidler and R. Urtasun, On Arxiv soon]



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[X. Chen, K. Kundu, Y. Zhu, S. Fidler and R. Urtasun, On Arxiv soon]

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Example 5: More Precise Grouping

• Given a single image, we want to infer **Instance-level Segmentation** and **Depth Ordering**



- Use deep convolutional nets to do both tasks simultaneously
- Trick: Encode both tasks with a single parameterization
- Run the conv. net at multiple resolutions
- Use MRF to form a single coherent explanation across all the image combining the conv nets at multiple resolutions
- Important: we do not use a single pixel-wise training example!

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Results on KITTI

[Z. Zhang, A. Schwing, S. Fidler and R. Urtasun, ICCV '15]



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More Results (including failures/difficulties)

[Z. Zhang, A. Schwing, S. Fidler and R. Urtasun, ICCV '15]



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Example 6: Enhancing freely-available maps



Toronto: Airport



Kyoto: Kinkakuji



San Francisco: Russian Hill



Sydney: At Harbour bridge

[G. Matthyus, S. Wang, S. Fidler and R. Urtasun, ICCV '15]



NYC: Times square



Monte Carlo: Casino

- Enhancing OpenStreetMaps
- Can be trained on a single image and test on the whole world
- Trick: Not to reason at the pixel level
- Very efficient: 0.1s/km of road
- Preserves topology and is state-of-the-art

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Example 7: Fashion

[E. Simo-Serra, S. Fidler, F. Moreno, R. Urtasun, CVPR15]



LOS ANGELES, CA 466 FANS 288 VOTES 62 FAVOURITES TAGS

CHIC EVERDAY FALL

COLOURS WHITE-BOOTS

NOVEMBER 10, 2014 GARMENTS

White Cheap Monday Boots Chilli Beans Sunglasses Missguided Romper Daniel Wellington Watch

COMMENTS

Nice!! Love the top! cute

Figure : An example of a post on http://www.chictopia.com. We crawled the site for 180K posts.



Figure 3: Visualization of the density of posts and fashionability by country.

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City Name	Posts	Fashionability
Manila	4269	6.627
Los Angeles	8275	6.265
Melbourne	1092	6.176
Montreal	1129	6.144
Paris	2118	6.070
Amsterdam	1111	6.059
Barcelona	1292	5.845
Toronto	1471	5.765
Bucharest	1385	5.667
New York	4984	5.514
London	3655	5.444
San Francisco	2880	5.392
Madrid	1747	5.371
Vancouver	1468	5.266
Jakarta	1156	4.398

Table 2: Fashionability of cities with at least 1000 posts.

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Figure : We ran a face detector that predicts also beauty of the face, age, ethnicity, mood.

• Face detector + attributes



http://www.rekognition.com



sunglasses : false (value : 0) eye_closed : open (value : 0) mouth_open_wide : 3% (value : 0.03) beauty : 99.42 (value : 0.99422) gender : female (value : 0)

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• Face detector + attributes



http://www.rekognition.com



confidence : true (value : 1) pose : roll(-6.26) , yav(-6.81) , pitch(1.66) race : white(0.99) emotion : happy:92%,confused:1% age : 60.9 (value : 60.9) smile : true (value : 0.87) glasses : no glass (value : 0.01) sunglasses : false (value : 0.03) beauty : 78.62 (value : 0.78628) gender : male (value : 1)

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August 21, 2015

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• Face detector + attributes



http://www.rekognition.com



confidence : true (value : 1) pose :rol(4.3) ,yaw(10.36),pitch(-5.4) race : white(0.73) emotion : happy:99%,calm:3% age : 29.12 (value : 29.12) smile : true (value : 20.86) glasses : no glass (value : 0) sunglasses : false (value : 0) eye_closed : open (value : 0) mouth_open_wide : 0% (value : 0) beauty : 53.67 (value : 0.30)

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Figure : Our model is a Conditional Random Field that uses many visual and textual features, as well as meta-data features such as where the user is from.
How Fashionable Are You?



Figure : We predict fashionability of users.



Figure : We predict what kind of outfit the person wears.

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How Fashionable Can You Become?



Current Outfit: Pink Outfit (3)

Recommendations: Heels (8) Pastel Shirts/Skirts (8) Black/Gray Tights/Sweater (5)



Current Outfit: Blue with Scarf (3)

Recommendations: Heels (8) Pastel Shirts/Skirts (8) Black Casual (8)





Recommendations: Black/Gray Tights/Sweater (5) Black Casual (5) Black Boots/Tights (5)

Current Outfit: Pink/Blue Shoes/Dress Shorts (3) Recommendations:

Black Casual (7) Black Heavy (3) Navy and Bags (3)



Current Outfit: Pink/Black Misc. (5)

Black Casual (8)



Recommendations: Pastel Dress (8) Black/Blue Going out (8)

Current Outfit: Formal Blue/Brown (5)



Figure : Examples of recommendations provided by our model. The parenthesis we show the fashionability scores.

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Not a big deal... but

• Appear all over the Tech and News

News and Tech websites							
NewScientist	QUARTZ Quartz	TECH TIMES	WIRCO.UK Wired, UK	Mashable Mashable			
AOL News (video)	THE HUFFINGTON POST	HUFFPOST STYLE CENTRAL Huffington Post, Canada	MSN, Canada	Protein Protein			
YAHOO! NEWS CANADA Yahoo, Canada	Science Daily Science Daily	Mail Online Daily Mail, UK	psfk PSFK	thestar.com			
gizmag Gizmag		iDigitalTimes iDigitalTimes					

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Not a big deal... but

- Appear all over the Tech and News
- All over the Fashion press

Fashion Magazines (Online)

ELLE BAZAAR marieclaire Marie Claire Harper's Bazaar **Red Magazine** Flle (UK) FASHION YAHOO! GLAMOUR COSMOPOLITAN STYLE Fashion Cosmopolitan Glamour Yahoo Style Magazine The Pasl The Pool (UK) FashionNotes

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Not a big deal... but

R. Urtasun (UofT)

• Appear all over the Tech and News

International News

- All over the Fashion press
- International News and TV (Fox, BBC, SkypeNews, RTVE, etc)

VOGUE Vogue (ES)	WIRD Wired (DE)	jetzt.de Stiddeutsche Zeitung Jetzt (DE)	Sinembargo.mx SinEmbargo (MX)	
Per Naukaw Polsce Nauka (PL)	STYLEBOOK Stylebook (DE)	marie claire Marie Claire (FR)	FASHION POLICENg Fashion Police (NG)	
Amsterdam Fashion (NL)	ANSAit Ansa (IT)	Pluska Pluska (SK)	E L L E	
IT NEWS IT News (SK)	POPSUGAR PopSugar (AU)	CenárioMT CenárioMT (BR)	Pressetext (AT)	
LaGazzetta dello Sport Edito insu @ Matavata La Gazzetta dello Sport (IT)	WOMAR Woman (ES)	CSIC (ES)	E FE: ESTILO EFE (ES)	> ≺ ≣ >
1	Deep Structured N	lodels	Augu	ıst 21, 2015

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Cosmopolitan (UK): The technology scores your facial attributes (this just keeps getting better, doesn't it) from your looks, to your age, and the emotion you're showing, before combining all the information using an equation SO complex we won't begin to go into it.

But the Most Important Impact



• Use the hinge loss to optimize the unaries only which are neural nets (Li and Zemel 14). Correlations between variables are not used for learning

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- (Domke 13) treat the problem as learning a set of logistic regressors
- Fields of experts (Roth et al. 05), not deep, use CD training
- Many ideas go back to (Boutou 91)

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Deep Structured Models

Conclusions:

- Modeling of correlations between variables
- Non-linear dependence on parameters
- Joint training of many convolutional neural networks
- Parallel implementation
- Wide range of applications: Word recognition, Tagging, Segmentation Future work:
 - Latent Variables
 - More applications

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