CSC321 T2

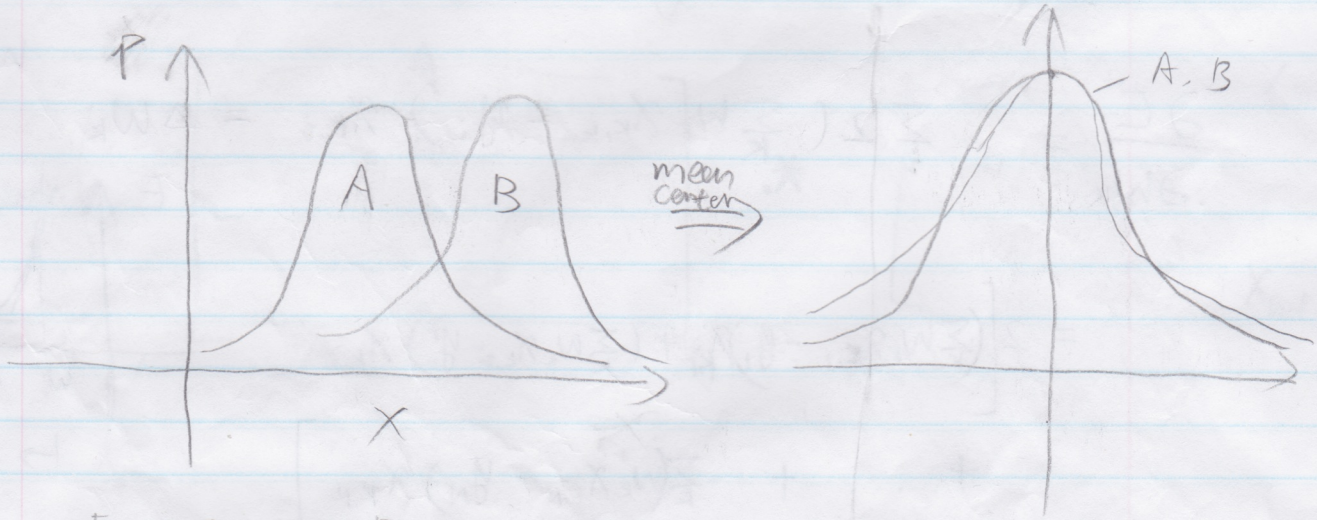
Mean-center (p 8)

$$X = \begin{bmatrix} x_{1,1} & x_{1,2} & x_{1,3} & \cdots & x_{1,N} \\ x_{2,1} & x_{2,2} & x_{2,3} & \cdots & x_{2,N} \\ \vdots & \vdots & & & \\ x_{D,1} & x_{D,2} & & & x_{D,N} \end{bmatrix}$$

$$\text{mean}(X,2) = \begin{bmatrix} \frac{1}{N}\sum_{i=1}^{N} x_{1,i} \\ \frac{1}{N}\sum_{i} x_{2,i} \\ \vdots \\ \frac{1}{N}\sum_{i} x_{D,i} \end{bmatrix} \xrightarrow{\text{repmat } N \text{ times}} = \overbrace{\begin{bmatrix} \frac{1}{N}\sum_{i} x_{1,i} & \cdots & \frac{1}{N}\sum_{i} x_{1,i} \\ \frac{1}{N}\sum_{i} x_{2,i} & \cdots & \frac{1}{N}\sum_{i} x_{2,i} \\ \vdots & & \\ \frac{1}{N}\sum_{i} x_{D,i} & \cdots & \frac{1}{N}\sum_{i} x_{D,i} \end{bmatrix}}^{N}$$

$$= M$$

$$X^* = X - M$$



$$A: \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad B: \begin{bmatrix} 10 \\ 11 \end{bmatrix} \quad \xrightarrow{mc} \quad A: \begin{bmatrix} -0.5 \\ 0.5 \end{bmatrix} \quad B: \begin{bmatrix} -0.5 \\ 0.5 \end{bmatrix}$$

Linear regression:

$$X = \begin{bmatrix} x_{1,1} & x_{1,2} & \cdots & x_{1,N} \\ x_{2,1} & x_{2,2} & \cdots & x_{2,N} \\ \vdots & \vdots & \ddots & \\ x_{D,1} & x_{D,2} & \cdots & x_{D,N} \end{bmatrix}$$

$$W = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_D \end{bmatrix} \qquad Y = \begin{bmatrix} y_1 & y_2 & \cdots & y_N \end{bmatrix}$$

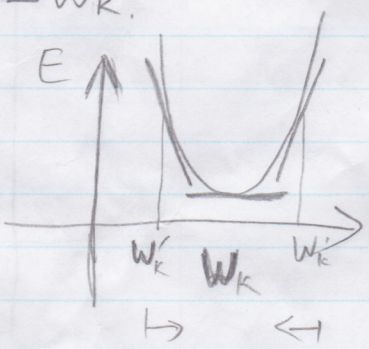
$$E = \sum_i \left(\sum_k w_k x_{k,i} - y_i \right)^2$$

$$= \sum_i \left(\begin{bmatrix} w_1 & w_2 & \cdots & w_D \end{bmatrix} \times \begin{bmatrix} x_{1,i} \\ x_{2,i} \\ \vdots \\ x_{D,i} \end{bmatrix} - y_i \right)^2$$

$$= \sum_i \left(\vec{W}^T \vec{X}_i + y_i \right)^2$$

$$w_k^* = w_k - \varepsilon \Delta$$

$$\frac{\partial E}{\partial w_k} = \sum_i 2 \left(\sum_k w_k x_{k,i} - y_i \right) x_{k,i} = \Delta w_k$$



$$= 2 \left[\left(\sum_k w_k x_{k,1} - y_1 \right) x_{k,1} + \left(\sum_k w_k x_{k,2} - y_2 \right) x_{k,2} \right.$$

$$\left. + \cdots + \sum_k \left(w_k x_{k,N} - y_N \right) x_{k,N} \right]$$

$$2 \, \text{sum} \left\{ \left(\left[\sum_k w_k x_{k,1} \quad \sum_k w_k x_{k,2} \quad \cdots \quad \sum_k w_k x_{k,N} \right] - \begin{bmatrix} y_1 & y_2 & \cdots & y_N \end{bmatrix} \right) .* \begin{bmatrix} x_{k,1} & x_{k,2} & \cdots & x_{k,N} \end{bmatrix} \right\} = 2 \, \text{sum} \left(\left(\begin{bmatrix} \vec{W}^T \vec{X}_1 & \vec{W}^T \vec{X}_2 & \cdots & \vec{W}^T \vec{X}_N \end{bmatrix} - \vec{Y} \right) .*$$

In Matlab:

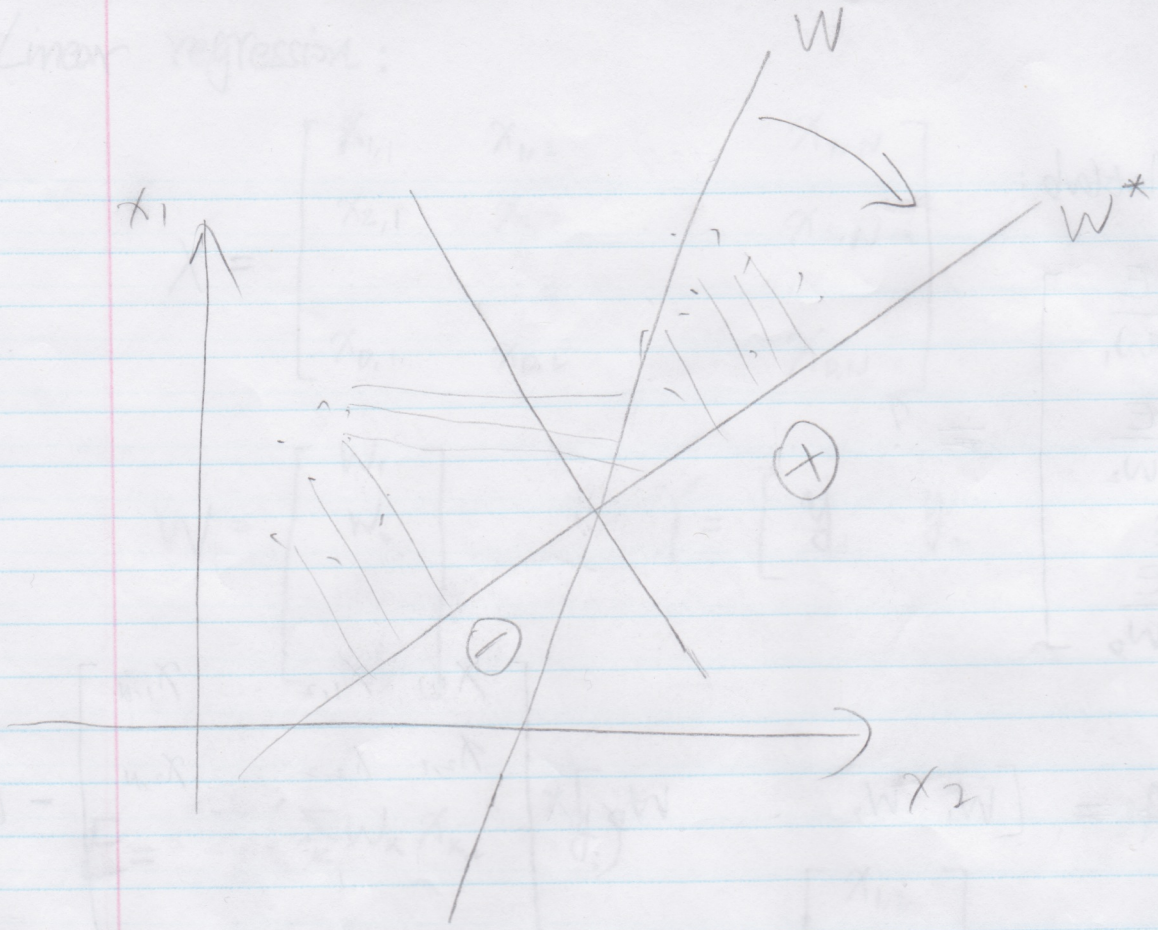
$$\begin{bmatrix} \frac{\partial E}{\partial w_1} \\ \frac{\partial E}{\partial w_2} \\ \vdots \\ \frac{\partial E}{\partial w_D} \end{bmatrix} = ?$$

$$A = [w_1 \; w_2 \; \dots \; w_D] \times \begin{bmatrix} x_{1,1} & x_{1,2} & \dots & x_{1,N} \\ x_{2,1} & x_{2,2} & \dots & x_{2,N} \\ \vdots & \vdots & \ddots & \\ x_{D,1} & x_{D,2} & & x_{D,N} \end{bmatrix} - [y_1 \; y_2 \dots$$

$$= \left[\sum_K w_K x_{K,1} - y_1 \;\; \sum_K w_K x_{K,2} - y_2 \dots \;\; \sum_K w_K x_{K,N} - y_N \right]$$

$\Downarrow$ repmat D row-wise

$$2 \; \text{sum}\left(\begin{bmatrix} A \\ A \\ \vdots \\ A \end{bmatrix} .* \begin{bmatrix} x_{1,1} & x_{1,2} & \dots & x_{1,N} \\ x_{2,1} & & & \\ & & & \\ & & & x_{D,N} \end{bmatrix} \right.$$


W
W*
x_1
x_2
+
−