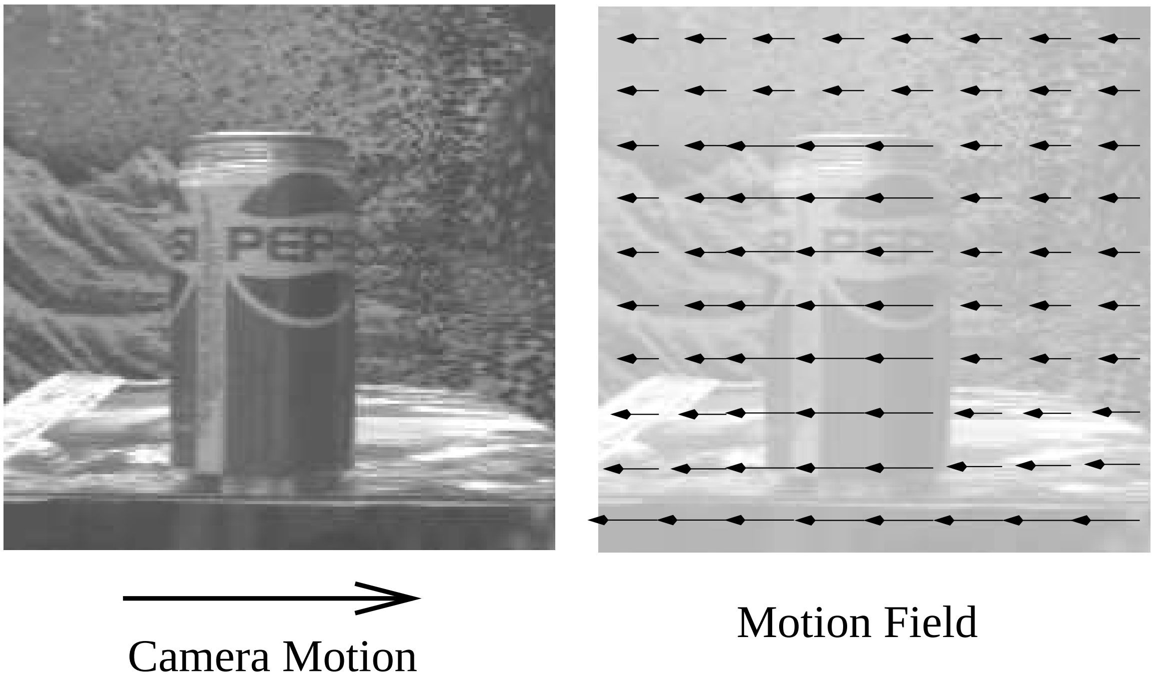
The Motion Field and Optical Flow

Motion Field: The image motion of fixed scene points.

Optical Flow: Approximation of the motion field obtained by tracking "stuff" (image texture) across two or more frames.



Difficulties in optical flow estimation:

- Variation in shading and reflections.
- Occlusion and transparency dictate a discontinuous or multiple valued flow field.
- Insufficient image texture for local 2D tracking.
- Deformation of image texture.
- Large displacements are common between successive frames.

Motion Constraint Equation

Brightness Constancy: Assume that the image intensity is constant along the image path $\vec{x}(\vec{p}, t)$, corresponding to a fixed point $\vec{p}$ on a visible surface. That is

$$\frac{d}{dt} I(\vec{x}(\vec{p}, t), t) = 0.$$

This is equivalent to the **motion constraint equation** (MCE)

$$\nabla I(\vec{x}, t) \cdot \vec{v}(\vec{x}, t) + \frac{\partial}{\partial t} I(\vec{x}, t) = 0.$$

Here $\nabla = (\frac{\partial}{\partial x}, \frac{\partial}{\partial y})$ is the spatial gradient operator. And $\vec{v}(\vec{x}, t) = \frac{\partial}{\partial t}\vec{x}(\vec{p}, t)$ is the motion field sampled at pixel $\vec{x} = \vec{x}(\vec{p}, t)$.

Phase Constancy: Replace $I(\vec{x}, t)$ above by the phase of a quadrature pair filter response.

Motion Constraint Line

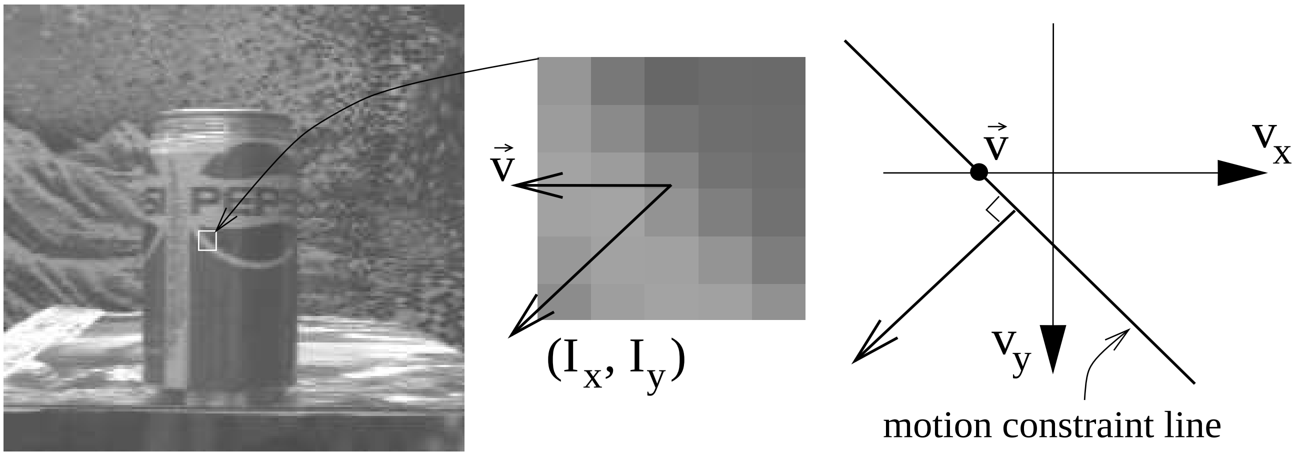
The motion constraint equation has the form:

$$\vec{c} \cdot (v_x, v_y, 1) = 0$$

Here

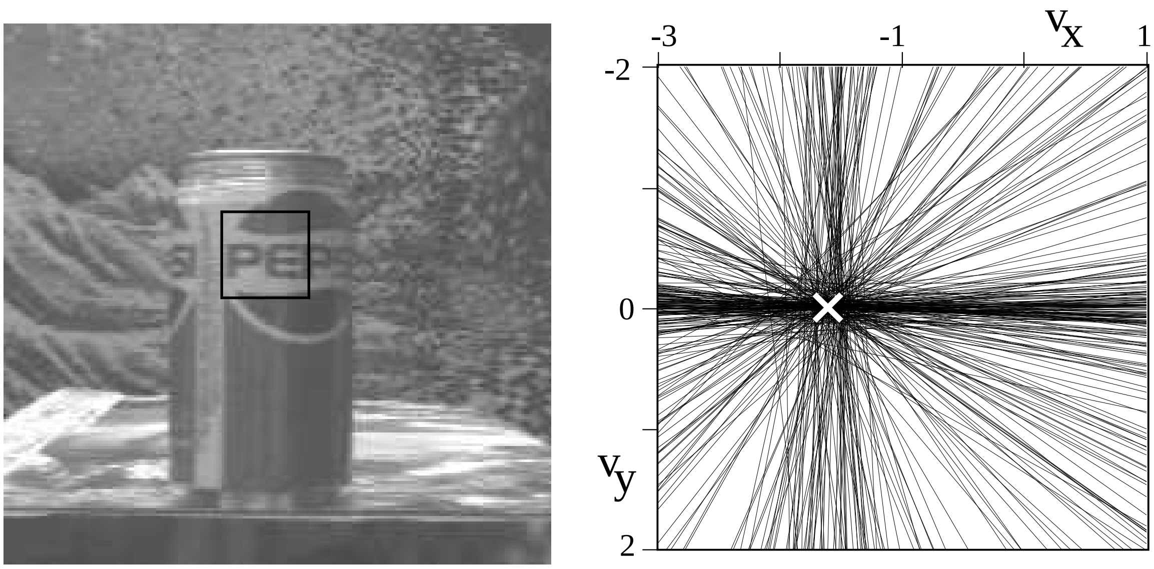
- $\vec{c} = (I_x(\vec{x}, t), I_y(\vec{x}, t), I_t(\vec{x}, t))^T$ can be obtained from image measurements.
- $\vec{v}(\vec{x}, t) = (v_x, v_y)^T$ is the unknown 2D image velocity at $(\vec{x}, t)$.

Geometrically, one such constraint determines a line in $\vec{v}$-space, perpendicular to $(I_x(\vec{x}, t), I_y(\vec{x}, t))$.

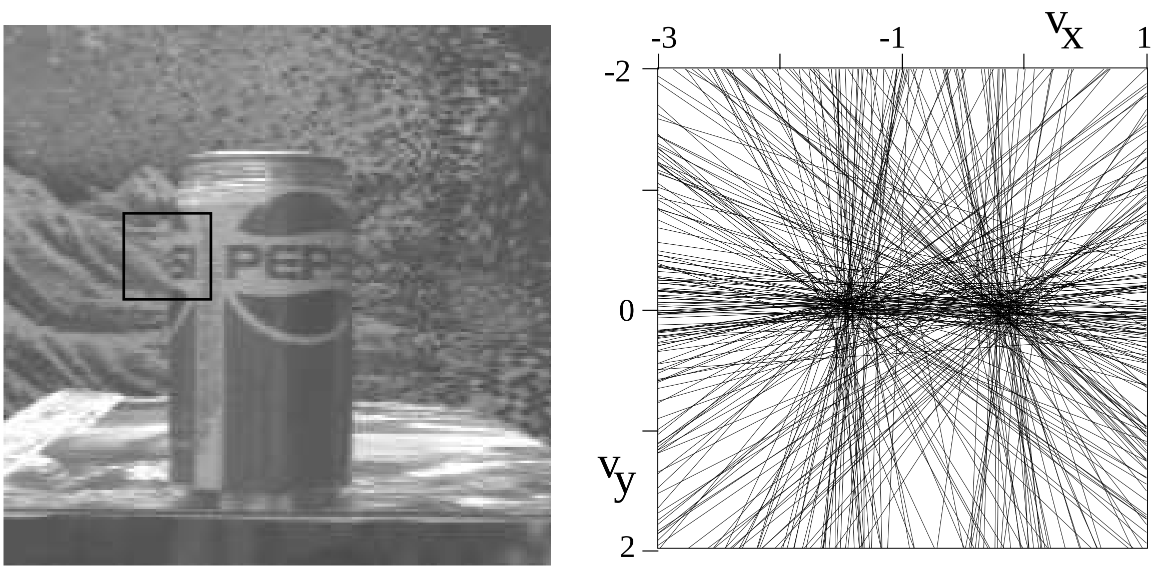


Motion Test Cases

1. Constant velocity within an image patch:



2. Occlusion boundary within an image patch:

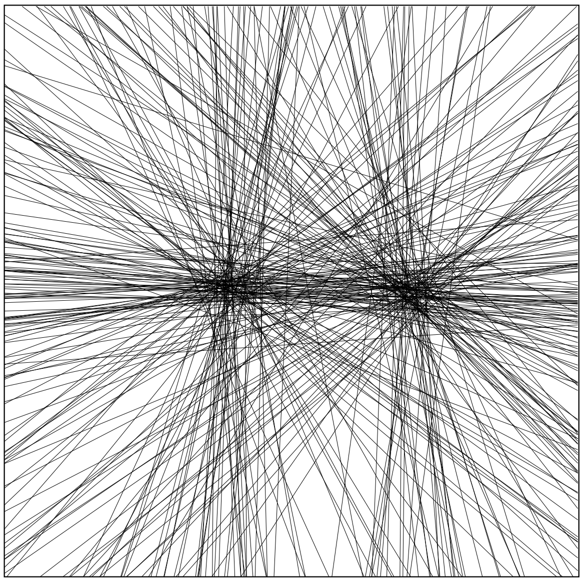


In case 2, least squares estimation (i.e. $\rho(e) = e^2$) is not appropriate.

The Aperture Problem

Computation of image motion requires the integration of several constraints from within an aperture.

Example: Patch contains an occlusion boundary:



Intersection of constraint lines

$$\vec{c}_k \cdot (v_x, v_y, 1) = 0$$

gives motion estimate(s).

Aperture Problem (Cont.)

The aperture must be chosen:

- sufficiently $\boxed{\text{LARGE}}$ to collect enough constraints,

and

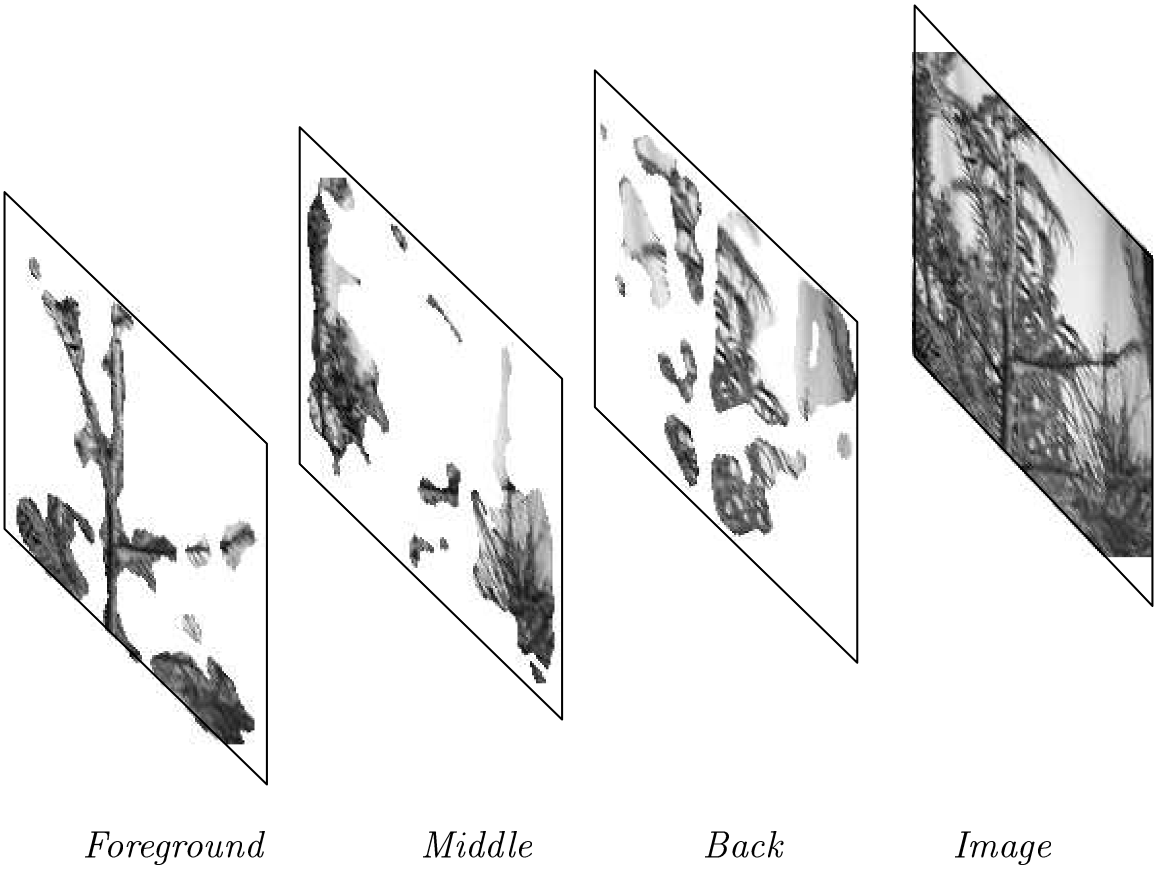
- sufficiently $\boxed{\text{small}}$ (with a suitable shape) to collect constraints from (predominantly) a single motion.

Sources of multiple motions:

- Occlusion boundaries
- Reflections
- Transparency
- Shadows

Fragmented Occlusion

Cannot expect to tune the aperture shape to complex fragmented regions.



Integration of Motion Constraints

Method for integrating motion constraints must accommodate multiple motions within a single aperture.

Possible approaches:

- Regularization with motion boundaries.

  (Black & Anandan – CVPR'91)

- Stochastic multi-layer segmentation

  (Darrell & Pentland – Motion Workshop '91)

- Robust regression and regularization.

  (Black & Anandan – ICCV'93)

- Mixture models

  (Jepson & Black – CVPR'93)

- Skin and bones

  (Ju, Black & Jepson – CVPR'97)