Belief Revision Theory
and
Knowledge Base Updates

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THE PROBLEM OF THEORY CHANGE

Knowledge base ψ

New fact μ

Revised/Updated Knowledge Base $\psi \circ \mu$.

What is $\psi \circ \mu$?

OUTLINE

1. Belief revision theory

2. Updates vs. revisions

3. Conditional reasoning

4. Complexity

5. Updates and queries

6. Time and change

7. The meta level

8. Current work

Applications:

- The view update problem
- Updating incomplete databases
- Updating logical databases
- Planning
- Non-monotonic logic
- Hypothetical reasoning
- Epistemology of science
- Legal systems

TWO APPROACHES

- Coherence Theories
  Make *minimal changes* necessary to maintain consistency. Example: AGM Theory.

- Justification Theories
  Keep track of justifications for beliefs and reject unjustified beliefs. Example: Truth Maintenance Systems.

The AGM Postulates

[Alchourrón, Gärdenfors and Makinson 1985]
[Katsuno and Mendelzon 1989]

Revision is a *function* mapping ψ and μ to $\psi \circ \mu$.

(R1) $\psi \circ \mu \vdash \mu$.

(R2) If $\psi \wedge \mu$ is satisfiable then $\psi \circ \mu \equiv \psi \wedge \mu$.

(R3) If μ is satisfiable then so is $\psi \circ \mu$.

(R4) If $\psi_1 \equiv \psi_2$ and $\mu_1 \equiv \mu_2$, then

$$\psi_1 \circ \mu_1 \equiv \psi_2 \circ \mu_2.$$

(R5) $\psi\circ(\mu_1\vee\mu_2) \equiv \psi\circ\mu_1$ or $\psi\circ(\mu_1\vee\mu_2) \equiv \psi\circ\mu_2$ or $\psi \circ (\mu_1 \vee \mu_2) \equiv (\psi \circ \mu_1 \vee \psi \circ \mu_2)$.

GIVING IT UP

$\psi \bullet \mu$ is the result of *contracting* μ from ψ.

Contraction Postulates

(C1) $\psi \vdash \psi \bullet \mu$.

(C2) If $\psi \nvdash \mu$, then $\psi \bullet \mu \equiv \psi$.

(C3) If $\nvdash \mu$, then $\psi \bullet \mu \nvdash \mu$.

(C4) If $\psi_1 \equiv \psi_2$ and $\mu_1 \equiv \mu_2$ then $\psi_1 \bullet \mu_1 \equiv \psi_2 \bullet \mu_2$.

(C5) $(\psi \bullet \mu) \wedge \mu \vdash \psi$.

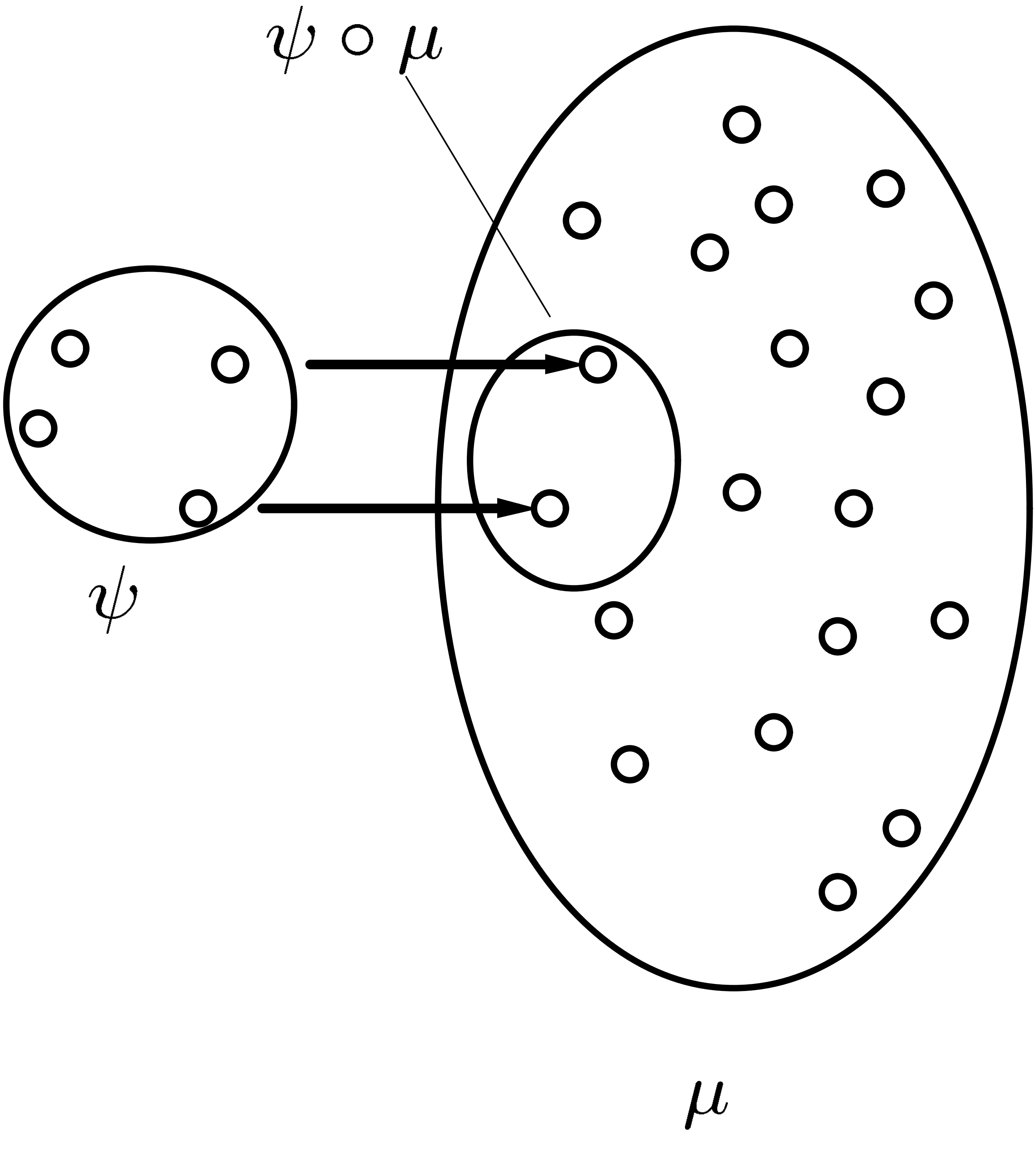
CONTRACTION AND REVISION

The Harper identity:

$$\psi \bullet \mu \equiv \psi \vee (\psi \circ \neg\mu)$$

The Levi identity:

$$\psi \circ \mu \equiv (\psi \bullet \neg\mu) \wedge \mu$$


$\psi \circ \mu$
ψ
μ

What are the "closest" worlds to ψ?
Define a *faithful pre-order* for ψ:

1. If $I, I' \in Mod(\psi)$ then $I <_\psi I'$ does not hold.

2. If $I \in Mod(\psi)$ and $I' \notin Mod(\psi)$ then $I <_\psi I'$.

3. If $\psi \equiv \phi$, then $\leq_\psi = \leq_\phi$.

The closest worlds to ψ are those that are *minimal* with respect to $<_\psi$.

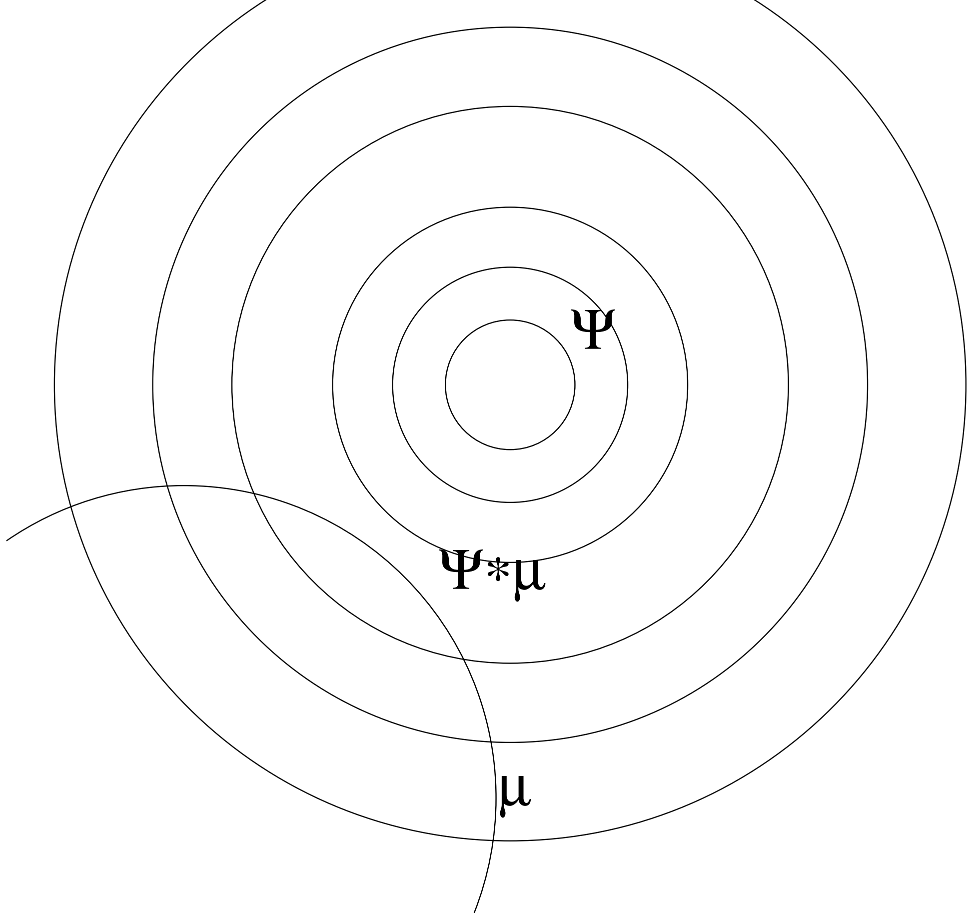
THEOREM
[Grove, 1988]
[Katsuno and Mendelzon, 1989]

Revision operator $\circ$ satisfies Conditions (R1)$\sim$(R5) if and only if for every KB ψ there exists a faithful total pre-order $\leq_\psi$ such that

$$\mathit{Mod}(\psi \circ \mu) = \mathit{Min}(\mathit{Mod}(\mu), \leq_\psi).$$

Proof:
If is straightforward.
Only-if: Given $circ$ and ψ, define $I \leq_\psi I'$ iff either $I \in \mathit{Mod}(\psi)$ or $I \in \mathit{Mod}(\psi \circ \mathit{form}(I, I'))$.

SYSTEM OF SPHERES



EXAMPLES OF REVISION OPERATORS

[Dalal 1988]

$dist(I, J) =$ number of propositional letters on which I and J differ.

$$dist(\mathit{Mod}(\psi), i) = min_{J \in \mathit{Mod}(\psi)} dist(J, I).$$

$I \leq_\psi J$ when

$$dist(\mathit{Mod}(\psi), I) \leq dist(\mathit{Mod}(\psi), J).$$

[Fagin, Ullman and Vardi 1983]

$\mathit{Mod}(\psi \circ \mu) = \bigcup_\phi \mathit{Mod}(\phi)$ where ϕ is a maximal subset of ψ consistent with μ.

When ψ is deductively closed, this yields $I \leq_\psi I'$ if either $I \in \mathit{Mod}(\psi)$ or neither one of I, I' is in $\mathit{Mod}(\psi)$.

THEOREM $\psi \circ \mu \equiv \mu$ [Alchourrón and Makinson, 1982]

Others:

Model Based:

Borgida(1985)
Weber(1986)
Satoh(1988)
Jackson and Pais (1991)

Formula Based:

Fagin, Ullman and Vardi(1983)
Ginsberg(1986)
Nebel(1989)
Forbus(1989)

See Gottlob and Eiter, PODS 1992.

[Winslett 1988]

POSSIBLE MODELS APPROACH

$dist(I, J)$ = set of propositional letters on which I and J differ.

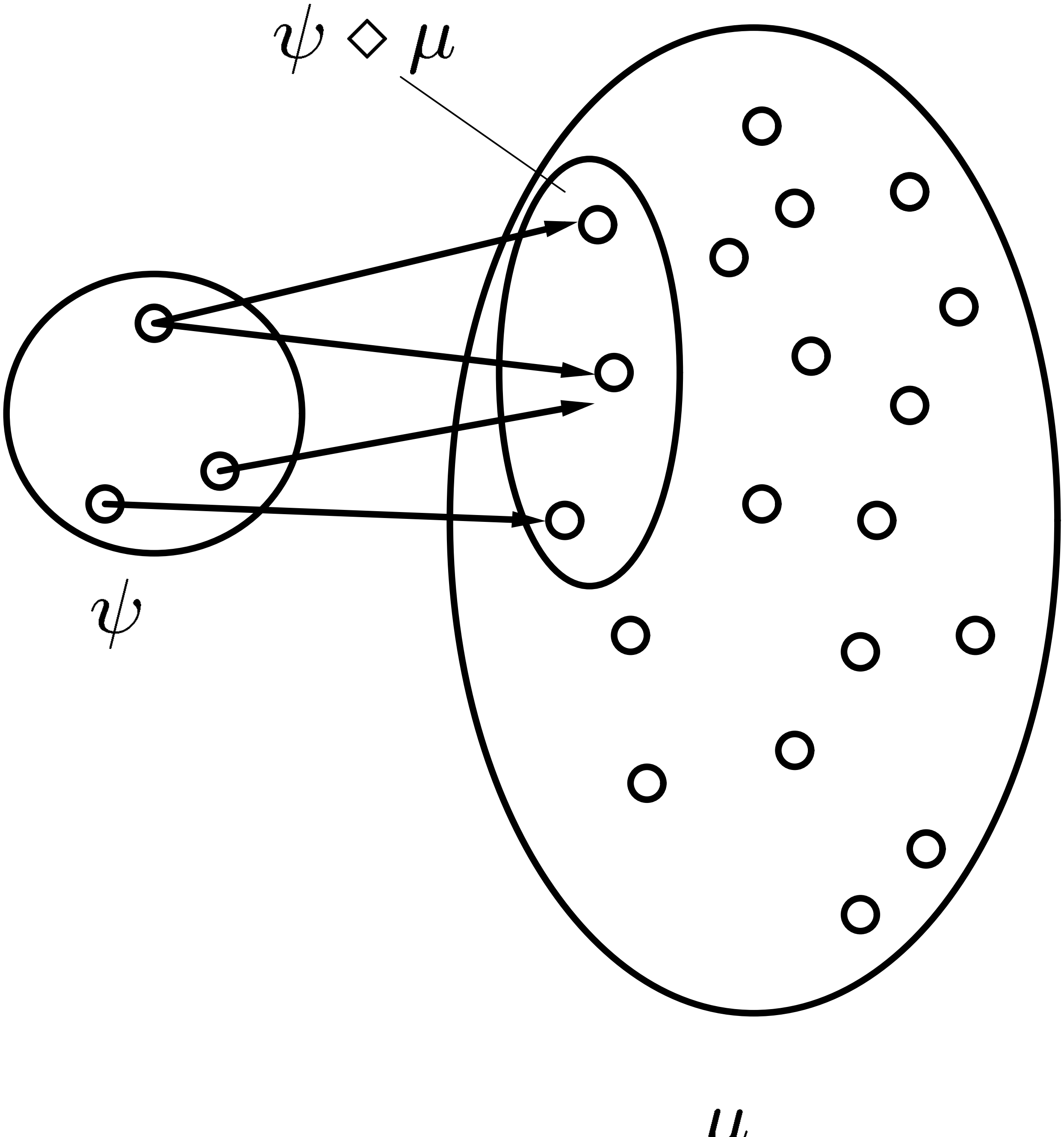
$J \leq_I K$ when

$$dist(J, I) \subseteq dist(K, J).$$

$$\mathit{Mod}(\psi \diamond \mu) = \bigcup_{I \in \mathit{Mod}(\psi)} \mathrm{Min}(\mathit{Mod}(\mu), I),$$

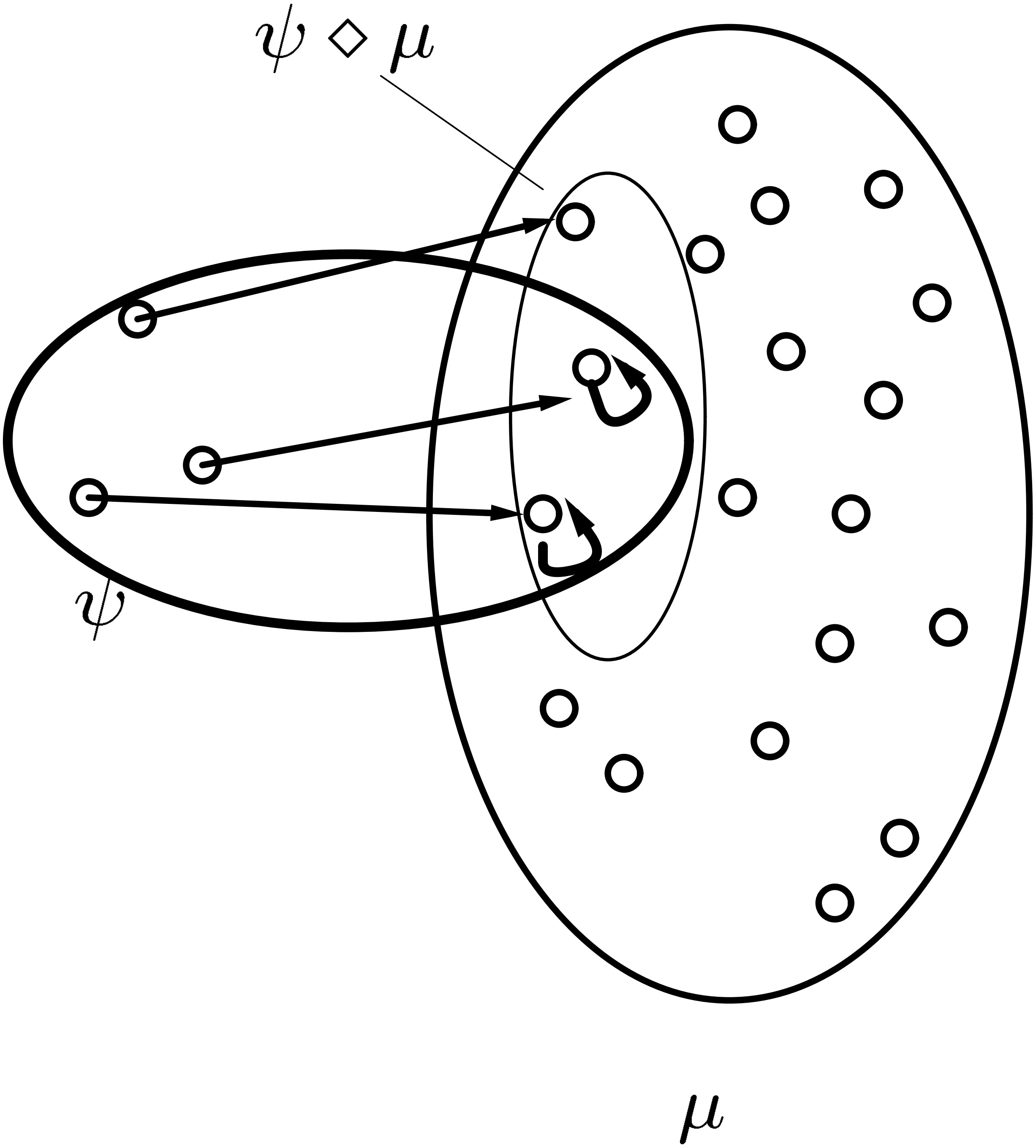
Surprise: $\diamond$ does not satisfy (R2)!

(R2) If $\psi \wedge \mu$ is satisfiable then $\psi \circ \mu \equiv \psi \wedge \mu$.



$\psi \diamond \mu$
ψ
μ

PMA VIOLATES (R2)



Example

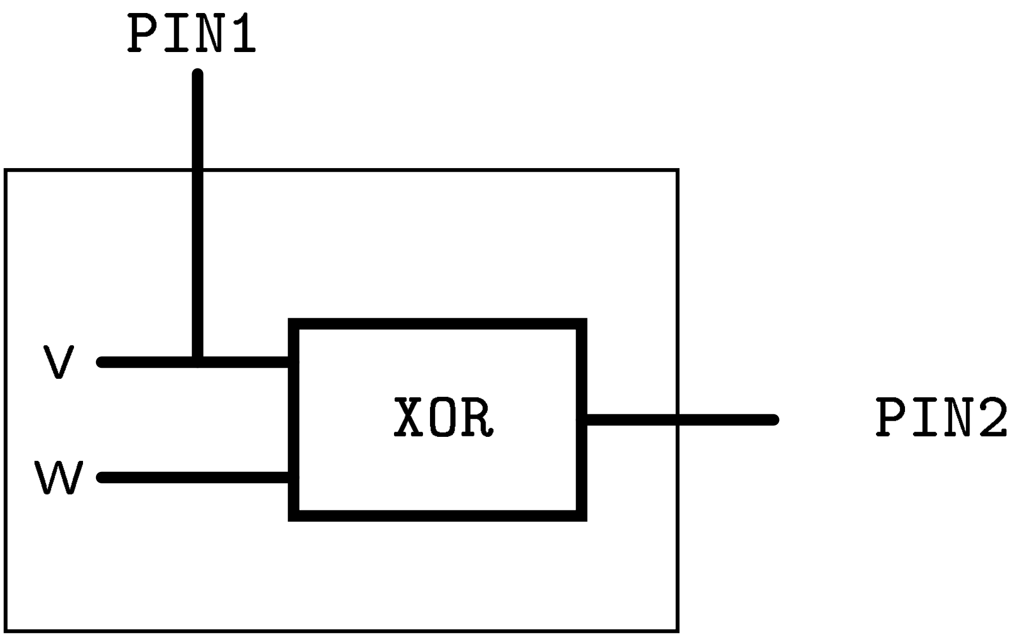
Vehicles V and W orbiting Venus.
The KB has two variables:

v	V has landed
w	W has landed

Garbled transmission: "`@#*%@ has landed.`"
KB $\psi = (v \wedge \neg w) \vee (\neg v \wedge w)$

Earth to V: `Land!`
V to Earth: `Done.`
New KB $= \psi \circ v = \psi \wedge v$ by (R2)
$\quad = v \wedge \neg w$
Desired KB $= v = \psi \diamond v$.

Example



Probe PIN2:	1
KB:	$\psi = (v \wedge \neg w) \vee (\neg v \wedge w)$
Probe PIN1:	1
New KB:	$= \psi \circ v = \psi \wedge v$ by (R2)
	$= v \wedge \neg w$

UPDATE VS. REVISION

Revision $\psi \circ \mu$: adjusting the KB's beliefs to new knowledge about a static world.

Update $\psi \circ \mu$: bringing the KB's beliefs up to date with changes in the world.

Revision satisfies (R2), update does not.

Cf. Keller and Winslett 1985, *change recording* vs. *knowledge adding*; Abiteboul and Grähne, 1985.

POSTULATES FOR UPDATE

Basic Postulates:

(U1) $\psi \diamond \mu \vdash \mu$.

(U2) If $\psi \vdash \mu$ then $\psi \diamond \mu \equiv \psi$.

(U3) If both ψ and μ are satisfiable then so is $\psi \diamond \mu$.

(U4) If $\psi_1 \equiv \psi_2$ and $\mu_1 \equiv \mu_2$, then

$$\psi_1 \diamond \mu_1 \equiv \psi_2 \diamond \mu_2.$$

Minimal Change Postulates:

(U5) $(\psi \diamond \mu) \wedge \phi \vdash \psi \diamond (\mu \wedge \phi)$.

(U6) If $\psi \diamond \mu_1 \vdash \mu_2$ and $\psi \diamond \mu_2 \vdash \mu_1$ then $\psi \diamond \mu_1 \equiv \psi \diamond \mu_2$.

(U7) If ψ is complete then $(\psi \diamond \mu_1) \wedge (\psi \diamond \mu_2) \vdash \psi \diamond (\mu_1 \vee \mu_2)$.

World Independence Postulate:

(U8) $(\psi_1 \vee \psi_2) \diamond \mu \equiv (\psi_1 \diamond \mu) \vee (\psi_2 \diamond \mu)$.

Define for each world I a *faithful* pre-order $\leq_I$:

- For any world J, if $I \neq J$ then $I \leq_I J$.

THEOREM

The update operator $\diamond$ satisfies Conditions (U1)$\sim$(U8). if and only if for each world I there exists a faithful partial pre-order $\leq_I$ such that

$$\mathit{Mod}(\psi \diamond \mu) = \bigcup_{I \in \mathit{Mod}(\psi)} \mathit{Min}(\mathit{Mod}(\mu), \leq_I).$$

CONDITIONAL REASONING

$\alpha > \beta$: "if α, then β."

What does it mean to believe $\alpha > \beta$?

Puzzling question. Example [Adams 1970]

"If Oswald did not shoot Kennedy, then someone else did."
"If Oswald had not shot Kennedy, then someone else would have."

The Ramsey Test:

$$\psi \vdash \alpha > \beta \Longleftrightarrow \psi \circ \alpha \vdash \beta.$$

A logic is *nontrivial* if there exist four sentences such that three of them are pairwise inconsistent and the fourth one is consistent with the other three.

Gärdenfors's Triviality Theorem: If $\circ$ is an AGM revision, the logic of ψ contains $>$ as a connective, and the Ramsey Test holds, then the logic is trivial.

What if we use update instead of revision?

An *update* version of the Ramsey Test:

$$\psi \vdash \alpha > \beta \Longleftrightarrow \psi \diamond \alpha \vdash \beta.$$

Grähne's Non-triviality Theorem: There is a non-trivial logic that contains $\diamond$ and $>$ and satisfies the Ramsey Test and the update postulates.

COMPUTATIONAL COMPLEXITY

What is the complexity of testing whether

$$\psi \circ \mu \vdash \nu \text{ ?}$$

What is the complexity of

$$\psi \vdash \mu > \nu \text{ ?}$$

Some data complexity results for propositional ψ, μ, ν: [Gottlob and Eiter 1992]

Dalal's revision is NP- and co-NP-hard, in $P^{NP[O(1)]}$; polynomial for Horn formulas.

Winslett's update is co-NP-complete; polynomial for Horn formulas.

TIME

- Revision: Incorporating new beliefs about a static world

- Update: Adjusting beliefs to changes in the world

Temporal KB: $\langle \psi, t \rangle$

$\texttt{TELL}(\langle \psi, t \rangle, \langle \beta, t \rangle) = \langle \psi \circ \beta, t \rangle.$
$\texttt{TELL}(\langle \psi, t \rangle, \langle \beta, t+1 \rangle) = \langle \psi \diamond \beta, t+1 \rangle.$

More general:

Temporal KB: $\langle \alpha_1, t_1 \rangle \ldots \langle \alpha_k, t_k \rangle$.

$\texttt{TELL}(\psi, \langle \beta, t_j \rangle) =$ replace α_j with $\alpha_j \circ \beta$.

Update is not needed!

If we know $\langle \alpha, t_1 \rangle$, what about $\langle \alpha, t_2 \rangle$?

To believe that things created by an incalculable series of causes can last forever is a serious mistake and is called the theory of permanency; but it is just as great a mistake to believe that things completely disappear; this is called the theory of non-existence.

The Teaching of Buddha

Temporal KB: $\psi = \{\langle \alpha_0, t_0 \rangle \ldots \langle \alpha_k, t_k \rangle\}$.

Let

$$\begin{aligned} \beta_0 &= \alpha_0 \\ \beta_i &= \beta_{i-1} \diamond \alpha_i \end{aligned}$$

$\texttt{TELL}(\psi, \langle \beta, t_j \rangle) =$ replace α_j with $\alpha_j \circ \beta$.

INSERTING FIRST-ORDER SENTENCES

[Grähne, Mendelzon, Revesz, 1992]

Inserting

$$E(xy) \;\leftarrow\; TR(xy)$$

$$TC(xz) \leftarrow\; TC(xy) \wedge E(yz) \;.\vee.\; E(xz)$$

$$TC(xz) \leftrightarrow\; TC(xy) \wedge TR(yz) \;.\vee.\; TR(xz)$$

into a graph produces the set $\{TR\}$ **of transitive reductions of the graph**.

COMPUTING UPDATES

Propositional language, possible models ordering:

$I_1 \leq_J I_2$ if diff$(I_1, J) \subset$ diff(I_2, J).

Membership in $Cn(\psi \diamond \mu)$ is at least coNP-hard.

Model Checking Approach: (cf. Halpern and Vardi, 1991)

Store all models of ψ explicitly in the KB. (Or keep ψ in DNF)

Then membership in $Cn(\psi \diamond \mu)$ has PTIME data complexity.

Agent thinks ψ

Agent has changed its mind to $\psi \circ \mu$

Syntax:

- $\mathtt{B}\phi$ "the agent believes ϕ."

- $\mathtt{O}\phi$ "the agent believes precisely ϕ."

Semantics:

- W is a set of possible worlds.

- $R \subseteq W \times W$ is an epistemic accessibility relation.

  $R_w = \{v \in W : wRv\}$

$$[\![\mathtt{B}\phi]\!] = \{w \in W : R_w \subseteq [\![\phi]\!]\}$$

$$[\![\mathtt{O}\phi]\!] = \{w \in W : R_w = [\![\phi]\!]\}$$

UPDATE MODELS

$$\mathcal{M} = \langle W, R, \leq, [\![ ]\!] \rangle,$$

- W set of worlds
- R_w epistemic accessibility relations
- $\leq_w$ comparative similarity orderings

THEOREM

[Grähne, Mendelzon, Reiter, 1992]. For every propositional belief revision system $\circ$ there is an update model $\mathcal{M}^{\oplus}$, such that for all sentences ϕ and ψ, we have

$$\psi \circ \mu \equiv \{\chi \in S : \mathcal{M}^{\oplus} \models \mathtt{0}\psi \diamond \mathtt{B}\mu \rightarrow \mathtt{B}\chi\}.$$

CURRENT WORK

- Knowledge base merging, arbitration (Revesz 1993)
- Update and reasoning about action (Del Val and Shoham, 1993)
- Changing temporal KB's
- Applications to schema evolution and object migration (Mendelzon and Milo, 1993)
- Update logics (Bonner and Kifer, 1992)