Semidefinite Programming Heuristics for Surface Reconstruction Ambiguities

Ady Ecker, Allan D. Jepson and Kiriakos N. Kutulakos
University of Toronto

Introduction

Several depth cues are locally ambiguous, e.g.

- Shape from shading [1]
- Shape from texture [2]
- Two-light photometric stereo
- Shape from defocus

We consider the problem of reconstructing a smooth surface under constraints that have discrete ambiguities.
Following [1], we convert these problems to a discrete optimization problem.
The problem is addressed using semidefinite programming (SPD) relaxation.
We improve the rounding phase using a combination of heuristics.

Formulation for two-fold ambiguities

Linear surface representation: $z(x,y) = \sum b_i(x,y)\, v_i$

General form: $\underset{v,d}{\operatorname{argmin}} \|\boldsymbol{A}v - \boldsymbol{B}d\|^2$, $v \in \mathbb{R}^m$, $d \in \{-1,1\}^n$

Adding smoothness term $\|\boldsymbol{E}v\|^2$: $\underset{v,d}{\operatorname{argmin}} \left\| \begin{bmatrix} \boldsymbol{E} \\ \boldsymbol{A} \end{bmatrix} v - \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{B} \end{bmatrix} d \right\|^2$

Eliminating continuous variables: $v = \boldsymbol{A}^{+}\boldsymbol{B}d$

Discrete optimization problem: $\underset{d}{\operatorname{argmin}} \|(\boldsymbol{A}\boldsymbol{A}^{+}\boldsymbol{B} - \boldsymbol{B})d\|^2$

$$\underset{X}{\operatorname{argmin}}\ \boldsymbol{C} \bullet \boldsymbol{X} = \sum \boldsymbol{C}_{ij}\, \boldsymbol{X}_{ij}$$

$$\boldsymbol{C} = (\boldsymbol{A}\boldsymbol{A}^{+}\boldsymbol{B} - \boldsymbol{B})^t(\boldsymbol{A}\boldsymbol{A}^{+}\boldsymbol{B} - \boldsymbol{B}), \boldsymbol{X} = dd^t$$

- Must solve a theoretically hard discrete optimization problem
- Global approach – local decisions affect the entire surface

Standard SDP approach

SDP relaxation: $\underset{X}{\operatorname{argmin}}\ \boldsymbol{C} \bullet \boldsymbol{X}$ s. t. $\boldsymbol{X}_{ii} = 1, \boldsymbol{X} \succcurlyeq 0$

Cholesky factorization: $\boldsymbol{X} = \boldsymbol{R}\boldsymbol{R}^t$

Embedding on the unit sphere: $u_i =$ i-th row of $\boldsymbol{R}$, $u_i \in \mathbb{R}^n$, $\|u_i\| = 1$

Goemans-Williamson rounding: $d_i = sign(u_i \cdot N)$, N is a random vector

Our SDP rounding heuristics

- Project the points embedded on the unit sphere onto planes in the subspace of their principle components
- Perform efficient circular sweeps [3]. Each sweep is $O(n^2)$
- Use Kernighan-Lin (K-L) local search for refinement

Example #1: Two-fold ambiguous normals

Texture cues provide two choices for the surface derivatives $p = \frac{dz}{dx}$, $q = \frac{dz}{dy}$
$(0, \dots 0, p_i, 0, \dots 0)d = a_{p_i} v$, $(0, \dots 0, q_i, 0, \dots 0)d = a_{q_i} v$

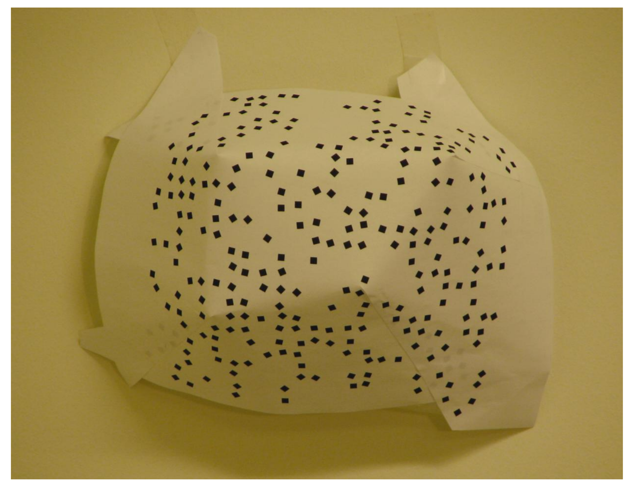
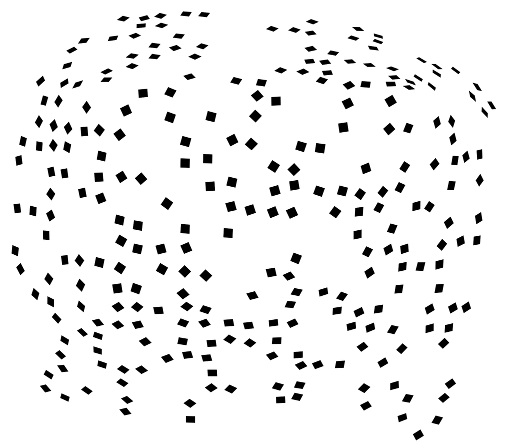
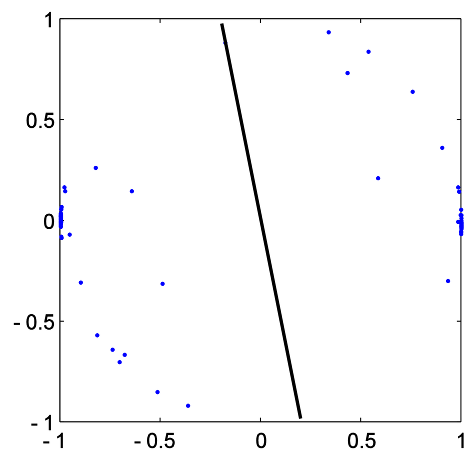
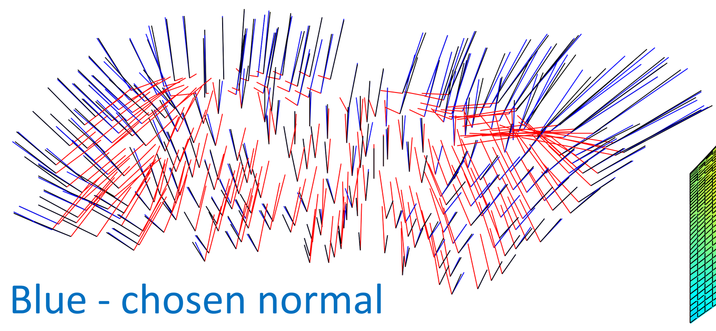
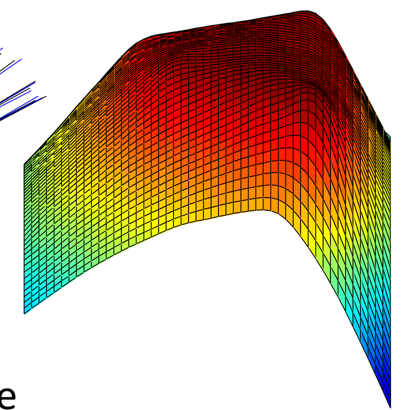


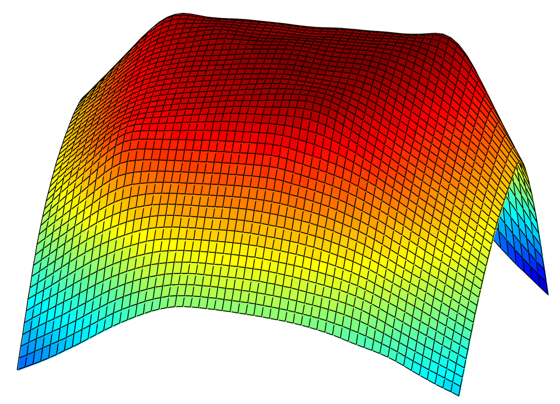
Image of textured surface


Extracted parallelograms


Circular sweep





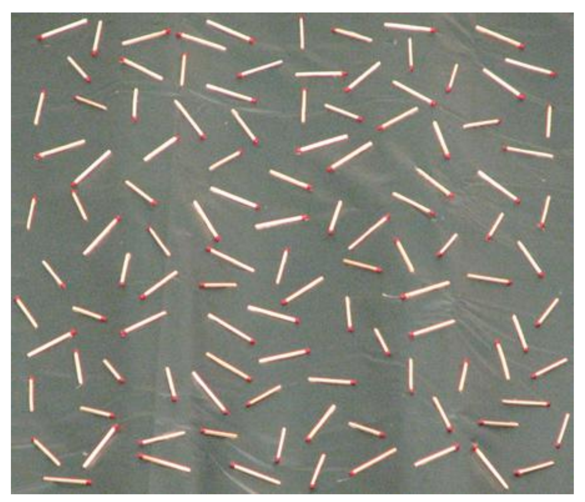


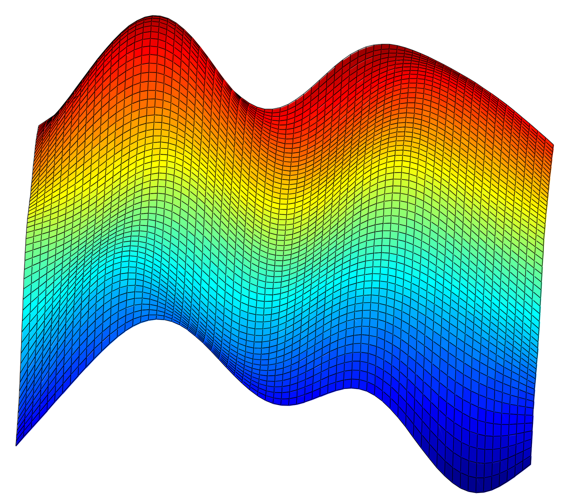
Blue - chosen normal
Red - alternative normal
Black - normal of the computed spline

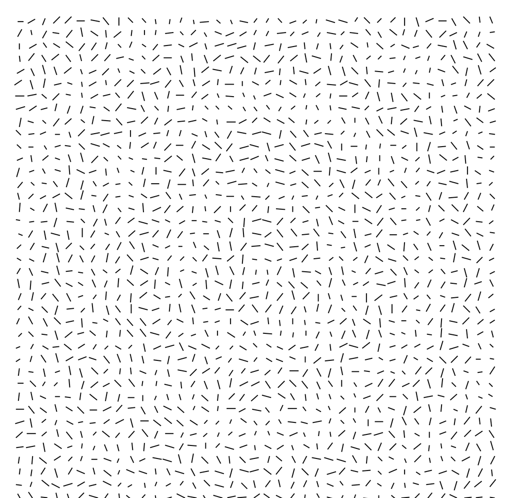
Example #2: Segments of known length

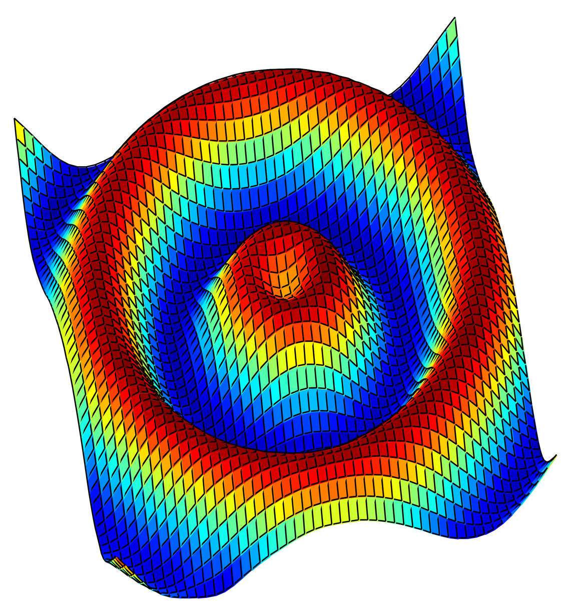
A segment of 3D length l and image length r_{ij} poses the constraint

$$z_i - z_j = (a_i - a_j)v = \left(0, \dots 0, \sqrt{l^2 - r_{ij}^2}, 0, \dots, 0\right) d$$



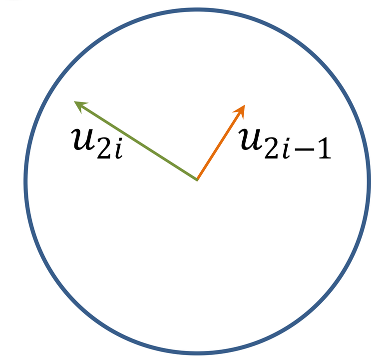






Example #3: Four-fold ambiguous normals

$p = d_1 p_1 + d_2 p_2$, $q = d_1 q_1 + d_2 q_2$
$d_1, d_2 \in \{-1,0,1\}$, $d_1 \cdot d_2 = 0$



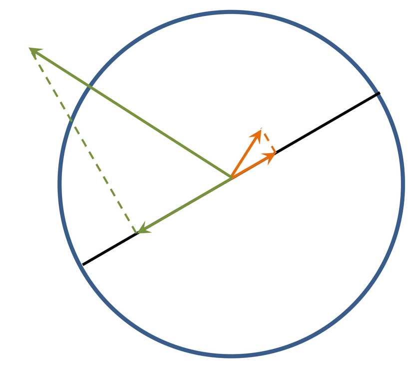
SDP relaxation

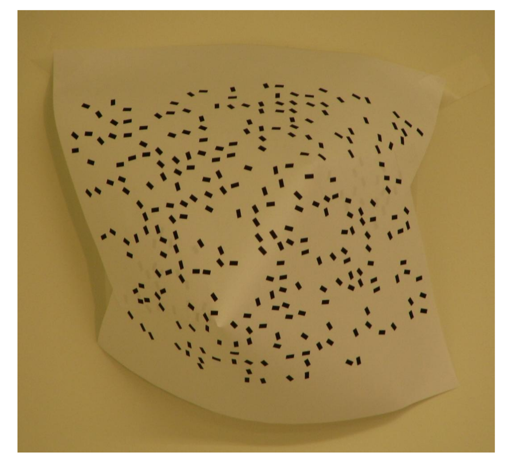
$$\underset{X}{\operatorname{argmin}}\ \boldsymbol{C} \bullet \boldsymbol{X} \quad \text{s. t.}$$
$$\boldsymbol{X} \succcurlyeq 0$$
$$\boldsymbol{X}_{2i-1,2i-1} + \boldsymbol{X}_{2i,2i} = 1$$
$$\boldsymbol{X}_{2i-1,2i} = 0$$

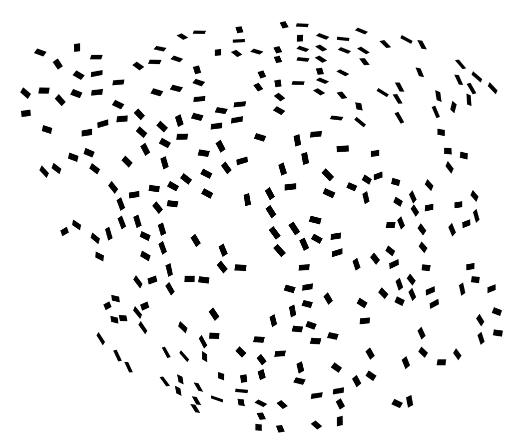
$\boldsymbol{X} = \boldsymbol{R}\boldsymbol{R}^T, \boldsymbol{X}_{ij} = u_i u_j^T$
$\|u_{2i-1}\|^2 + \|u_{2i}\|^2 = \boldsymbol{1}$
$u_{2i-1} \cdot u_{2i} = \boldsymbol{0}$

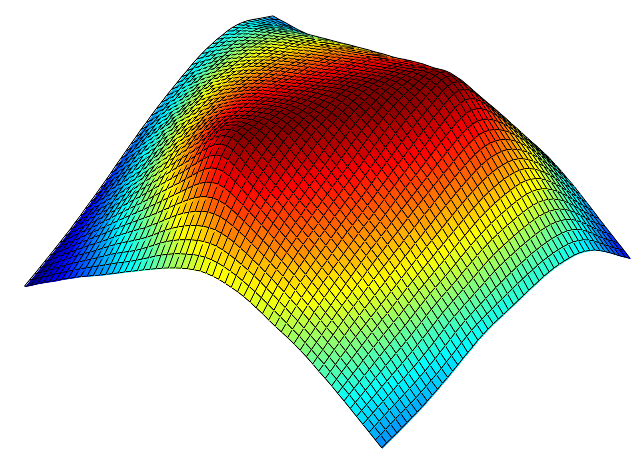
Rounding heuristics

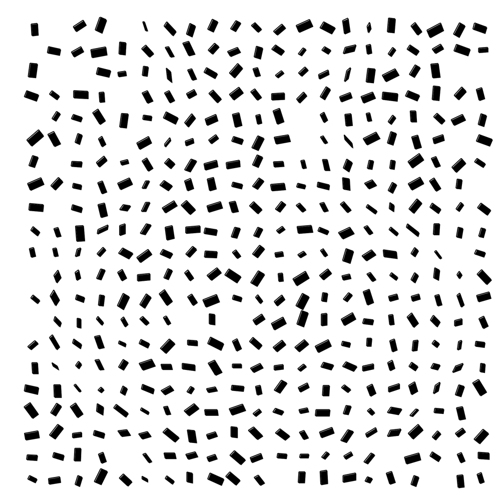
- Scale u_i by the magnitude of the i-th column of $\boldsymbol{X}$
- Circular sweeps – for each pair of vectors, the smaller projection is considered inactive
- Count the percentage p_i each variable is inactive, and modify the diagonal $\boldsymbol{C}_{ii} = \boldsymbol{C}_{ii} + \mu p_i$
- Solve the modified SDP
- Use K-L for refinement

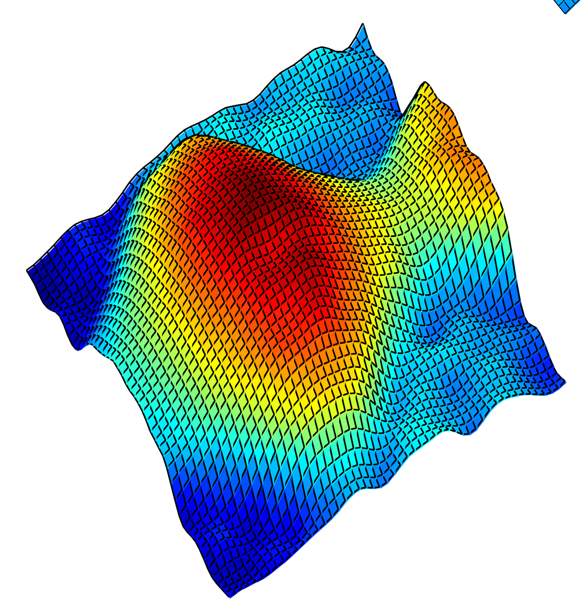












References

1. Zhu, Q., Shi, J.: Shape from shading: Recognizing the mountains through a global view. In: Proc. CVPR 2006, pp. 1839–1846.
2. Forsyth, D.: Shape from texture and integrability. In: Proc. ICCV 2001, pp. 447–452.
3. Burer, S., Monteiro, R.D.C., Zhang, Y.: Rank-two relaxation heuristics for max-cut and other binary quadratic programs. SIAM J. on Optimization 12(2), pp. 503–521, 2002.