Let's make another instance of our framework!

Very Busy Expressions

$$\texttt{if [a>b]}^1 \texttt{ then ([x:= b-a]}^2\texttt{;[y:= a-b]}^3\texttt{) else ([y:= b-a]}^4\texttt{;[x:= a-b]}^5\texttt{)}$$

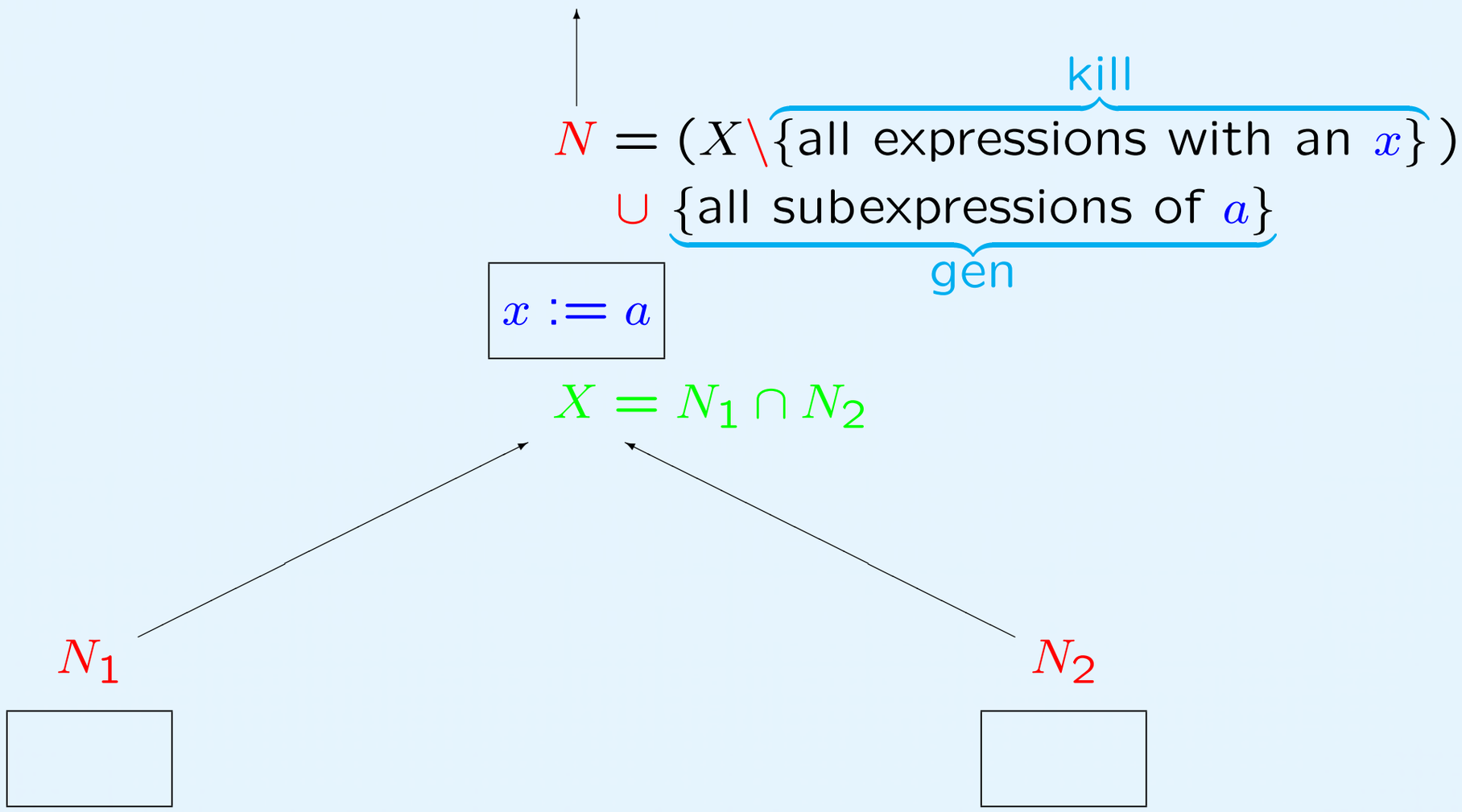
$$\texttt{[t1:= b-a]}^A\texttt{;[t2:= b-a]}^B\texttt{;}$$
$$\texttt{if [a>b]}^1\texttt{ then ([x:=t1]}^2\texttt{;[y:=t2]}^3\texttt{) else ([y:=t1]}^4\texttt{;[x:=t2]}^5\texttt{)}$$

An expression is very busy at the exit from a label if, no matter what path is taken from the label, the expression is always used before any of the variables occurring in it are redefined.

Check List

- Define the semi-lattice: dataflow facts and how to combine them!
    - Decide on the direction of the analysis: forward vs backward.
    - Sanity check: the corresponding order should make sense!
    - Decide on the initial values.
- Design the transfer functions:
    - How does each statement affect the dataflow facts?
    - Sanity check: Monotonicity!

The Design Process



Very Busy Expressions: Formal Setup

Dataflow Facts: $D = \mathcal{P}(Exp)$

Domain: complete meet semi-lattice $(D, \bigcap, D)$

unlike live variables: here we want the greatest fixed point!

Direction: Backward

Transfer Functions:

$$VB_\bullet(l) = \begin{cases} \emptyset & l = exit \\ \bigcap_{(l,l') \in flow} VB_\bullet(l') & \text{otherwise} \end{cases}$$

$$VB_\circ(l) = RD_\bullet(l) \setminus \{exp | var(exp) \cap write(l) \neq \emptyset\} \cup computed(l)$$

Reaching Definitions

	Live Variables	Reaching Definitions	Very Busy Expressions
$\mathbf{L}$	$\mathcal{P}(Var)$	$\mathcal{P}(Var \times Loc)$	$\mathcal{P}(Exp)$
$\sqcap$	$\cup$	$\cup$	$\cap$
$\sqsubseteq$	$\supseteq$	$\supseteq$	$\subseteq$
$\bot$	$\emptyset$	$\emptyset$	Exp
	Backward	Forward	Backward

Reaching Definitions

$[y := x]^1$; ⟵ $\{(x,?),(y,?),(z,?)\}$

$[z := 1]^2$; ⟵ $\{(x,?),(y,1),(z,?)\}$

while $[y > 0]^3$ **do** ⟵ $\{(x,?),(y,1),(y,5),(z,2),(z,4)\}$

  $[z := z * y]^4$; ⟵ $\{(x,?),(y,1),(y,5),(z,2),(z,4)\}$

  $[y := y - 1]^5$ ⟵ $\{(x,?),(y,5),(z,4)\}$

od; ⟵ $\{(x,?),(y,1),(y,5),(z,2),(z,4)\}$

$[y := 0]^6$ ⟵ $\{(x,?),(y,6),(z,2),(z,4)\}$

Reaching Definitions: Formal Setup

Dataflow Facts: $D = Vars \times Locs$

Domain: $(D, \cup, \emptyset, D)$

Direction: Forward

Transfer Functions:

$$RD_\circ(l) = \begin{cases} Var \times \{?\} & l = init \\ \bigcup_{(l',l) \in flow} RD_\bullet(l') & \text{otherwise} \end{cases}$$

$$RD_\bullet(l) = RD_\circ(l) \setminus write(l) \times Locs \cup write(l) \times \{l\}$$