CSC410

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Model Checking

Overview

- Nontraditional use of nontraditional logic!
- Checking whether a formula is satisfied in a finite domain.
  - Model: finite-state transition system
  - Logic: Propositional Temporal Logic.
  - Verification Procedure: exhaustively search of the state space to determine the truth of specification.

Why Model checking?

- Doesn't aim too high!
  - Originally restricted to finite-state systems.
  - applicable to systems with "short" descriptions.
  - control-oriented systems such as hardware, protocols, ...
- Fully automatic with low computational complexity.
- Can be viewed as an elaborate debugging tool: counterexamples.

First Step:
We need a formal model!

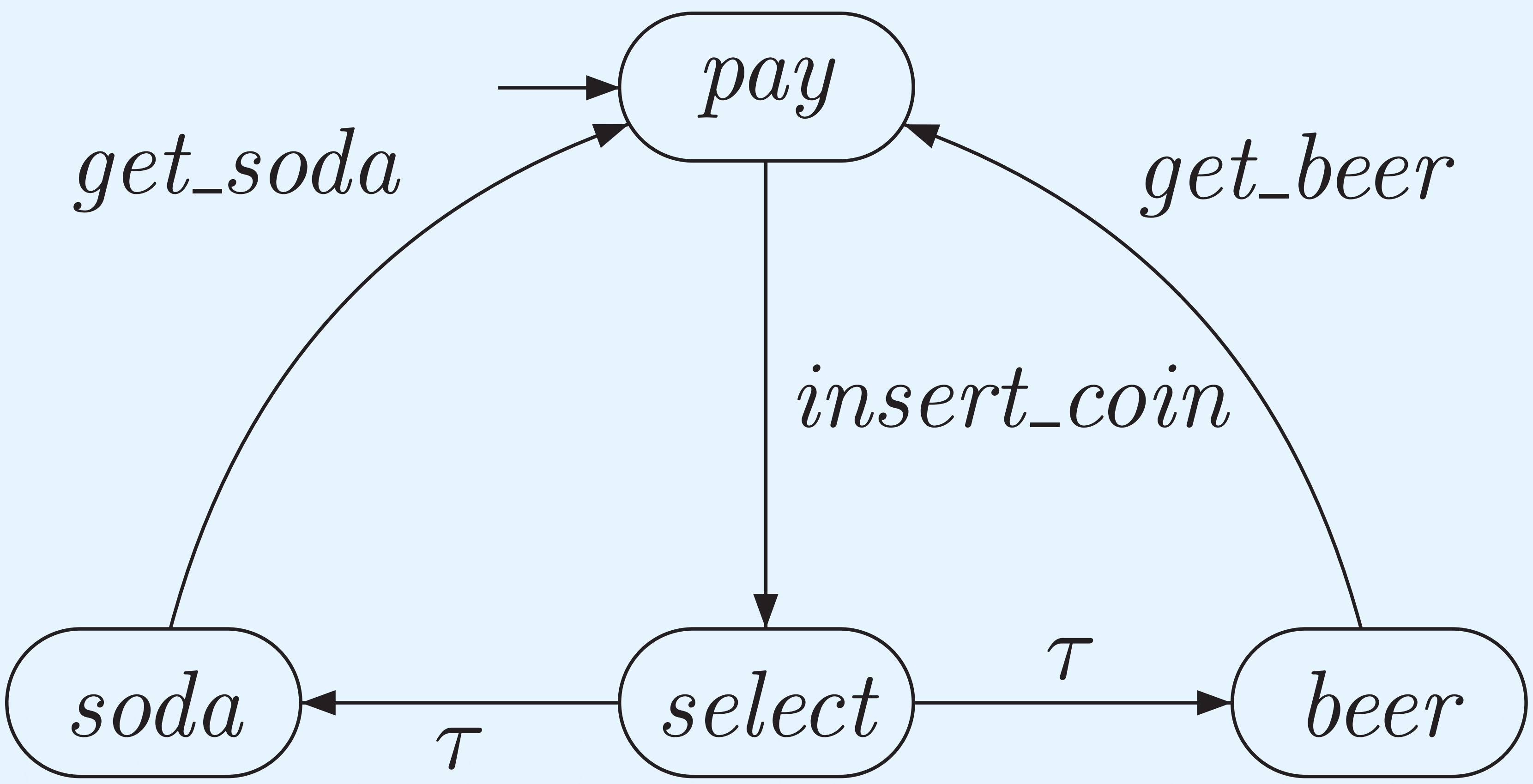
Labeled Transition Systems

A *transition system* TS is a tuple $(S, Act, \rightarrow, I, AP, L)$ where

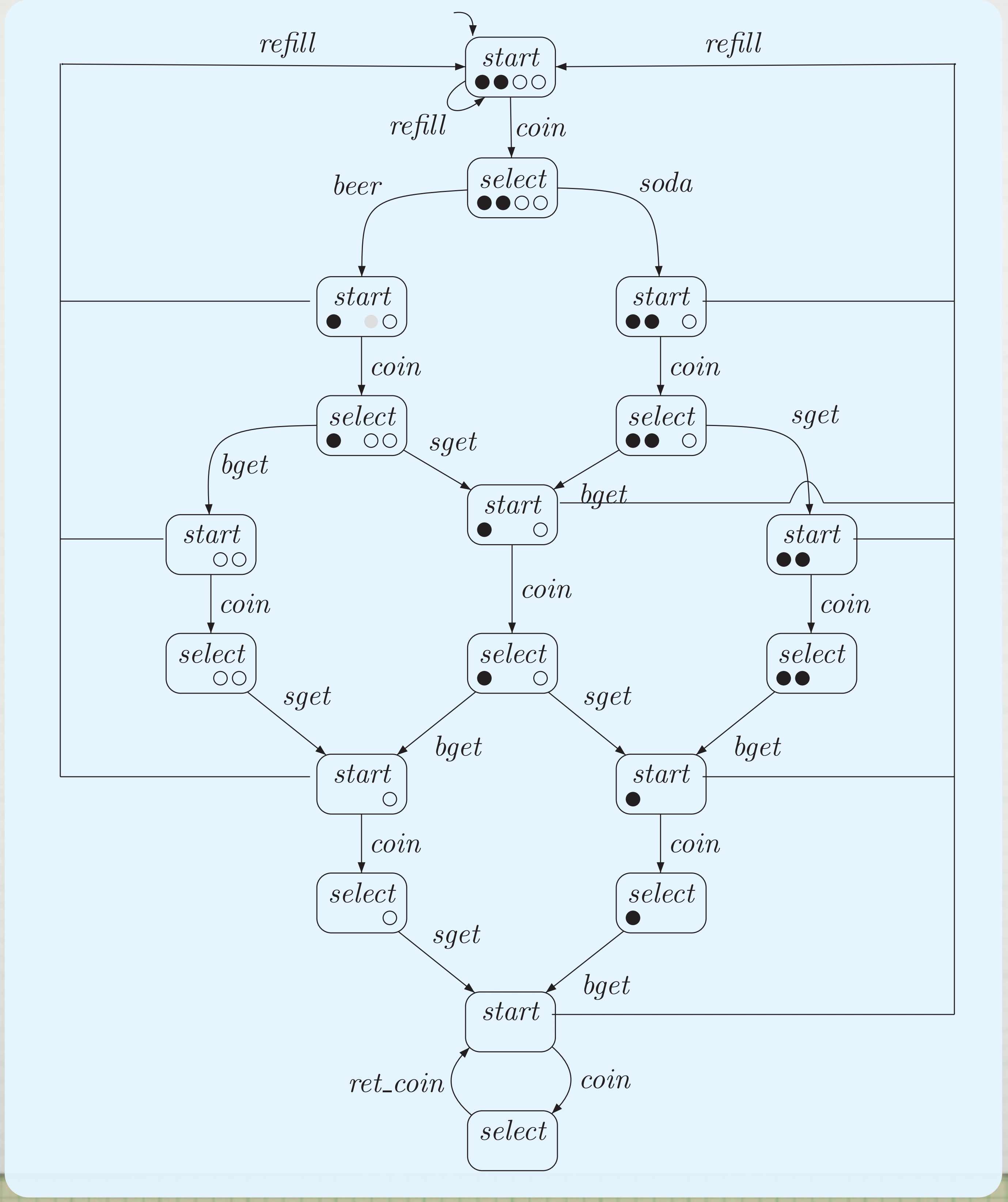
- S is a set of states,
- Act is a set of actions,
- $\longrightarrow \subseteq \ S \times Act \times S$ is a transition relation,
- $I \subseteq S$ is a set of initial states,
- AP is a set of atomic propositions, and
- $L : S \rightarrow 2^{AP}$ is a labeling function.

TS is called *finite* if S, Act, and AP are finite.

Example



Example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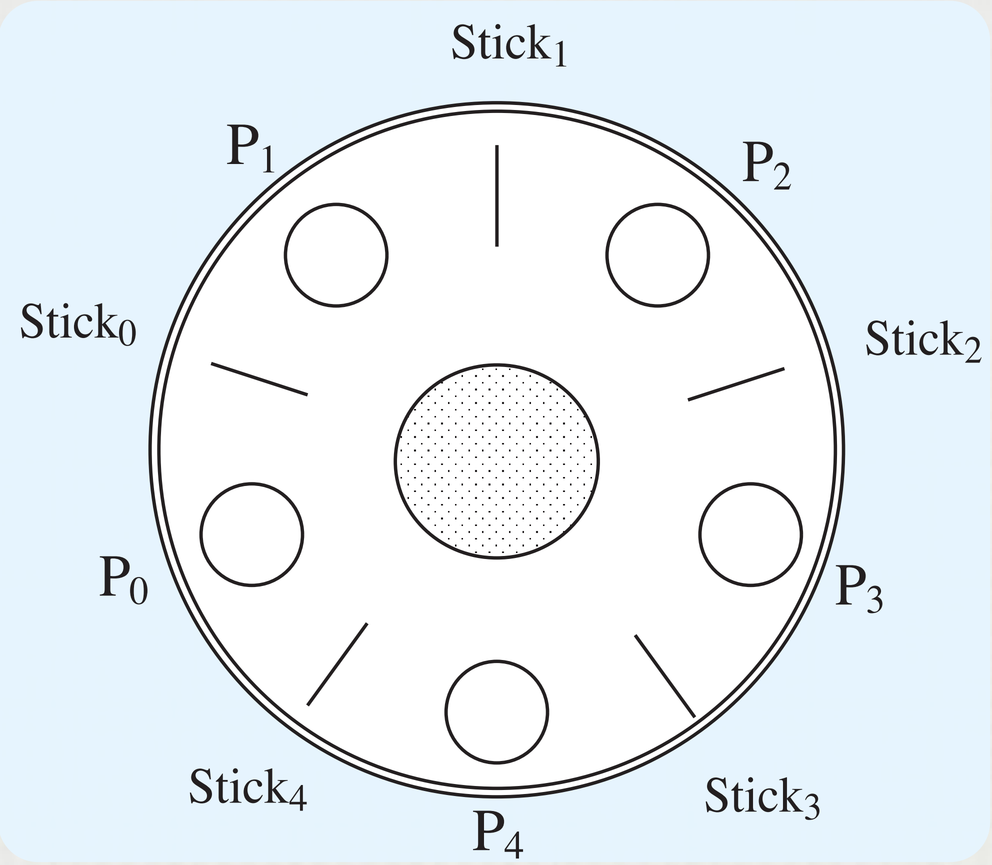
Second Step:
We need a formal Specification!

Example: Dining Philosophers



There are 5 philosophers at a table sharing 5 chopsticks for eating.

Each philosopher needs two chopsticks to eat.

At each point in time at most one of two neighbouring philosophers can eat.

Classic deadlock scenario example!

Reachability

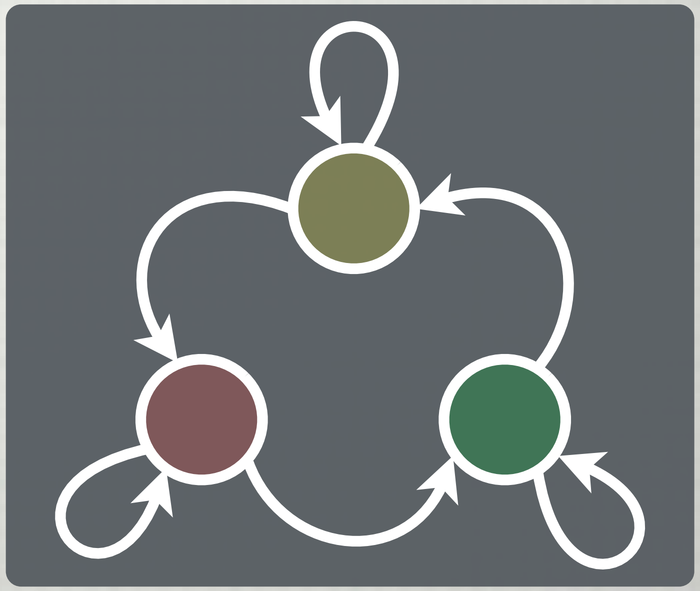
Problem: given an TS, and a target set T, is T reachable from Q_0.

Solution? Depth First Search, in O(n+m) time.

What if we are interested in more sophisticated properties?

Suggest a non-reachability property for philosophers!

The light will always eventually turn green.



Option 1 for properties beyond reachability ...

One TS as a Spec for Another TS!

Given a TS M for the model and a TS S for the specification:

Question: Is every behaviour of M a behaviour of S?

$$L(M) \subseteq L(S)$$

Solvable in PSpace: linear in M and exponential in S.

Best choice: new logic!

Alternative: Temporal Logic

- Language for describing properties of infinite sequences.
- Extension of propositional logic.
- Uses temporal operators to describe sequencing properties.

Linear Temporal Logic

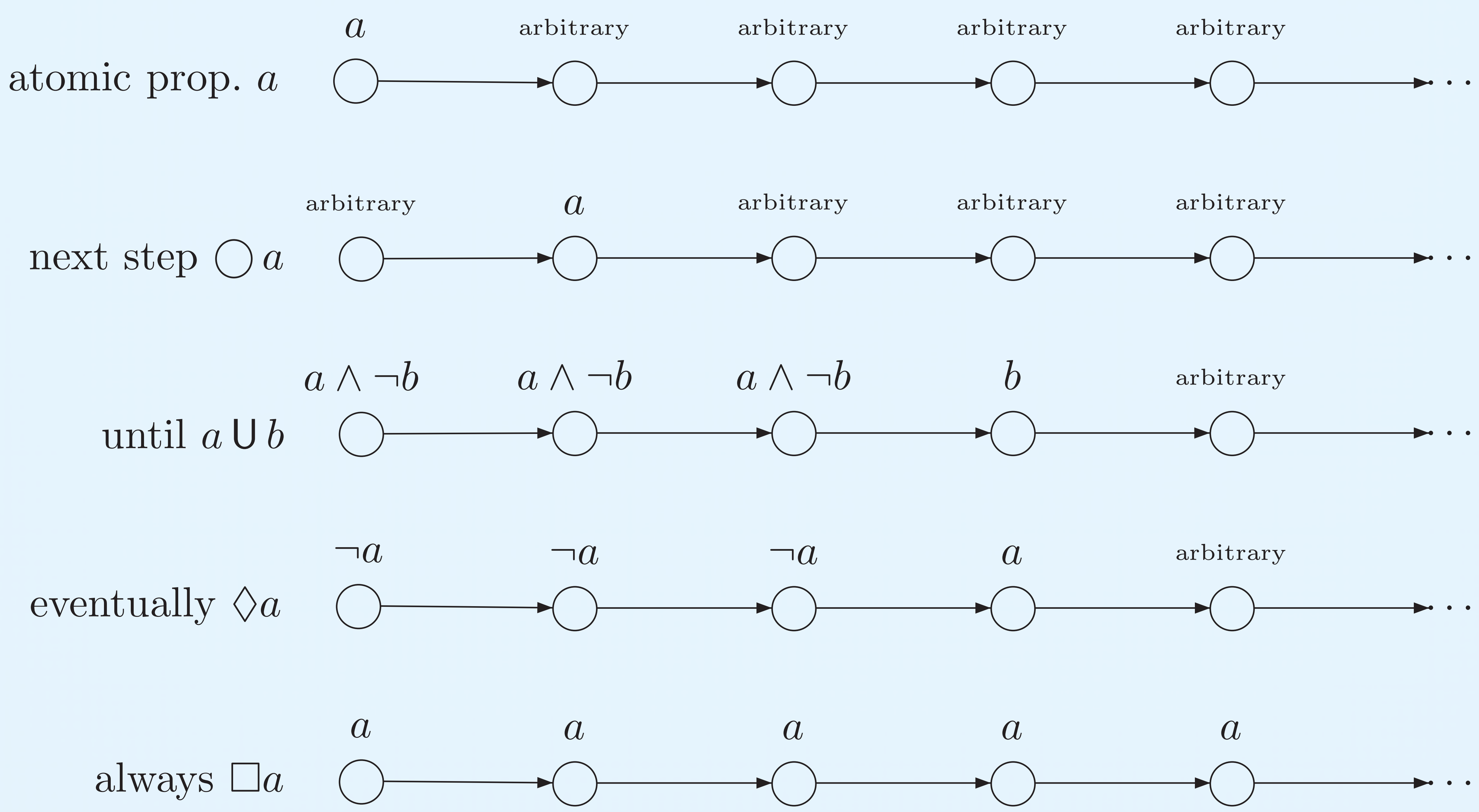
LTL Syntax

$$\varphi ::= \text{true} \;\Big|\; a \;\Big|\; \varphi_1 \wedge \varphi_2 \;\Big|\; \neg\varphi \;\Big|\; \boxed{\bigcirc\, \varphi \;\Big|\; \varphi_1 \,\mathsf{U}\, \varphi_2}$$

$a \in AP$

$$\Diamond\varphi \overset{\text{def}}{=} \text{true}\,\mathsf{U}\,\varphi \qquad\qquad \Box\varphi \overset{\text{def}}{=} \neg\Diamond\neg\varphi$$

LTL: Intuition



LTL Semantics

LTL is interpreted over paths.

These paths are (infinite) words labeled with subset of the atomic propositions (AP) that are true at each letter.

$$
\begin{array}{llll}
\sigma & \models & \text{true} & \\
\sigma & \models & a & \text{iff} \quad a \in A_0 \quad (\text{i.e., } A_0 \models a) \\
\sigma & \models & \varphi_1 \wedge \varphi_2 & \text{iff} \quad \sigma \models \varphi_1 \text{ and } \sigma \models \varphi_2 \\
\sigma & \models & \neg\varphi & \text{iff} \quad \sigma \not\models \varphi \\
\sigma & \models & \bigcirc\varphi & \text{iff} \quad \sigma[1\ldots] = A_1 A_2 A_3 \ldots \models \varphi \\
\sigma & \models & \varphi_1 \,\mathsf{U}\, \varphi_2 & \text{iff} \quad \exists j \geqslant 0.\ \sigma[j\ldots] \models \varphi_2 \ \text{ and } \ \sigma[i\ldots] \models \varphi_1, \text{ for all } 0 \leqslant i < j
\end{array}
$$

LTL's $\models$ is the smallest relation satisfying the above rules.

$$
\begin{array}{llll}
\sigma & \models & \Diamond\varphi & \text{iff} \quad \exists j \geqslant 0.\ \sigma[j\ldots] \models \varphi \\
\sigma & \models & \Box\varphi & \text{iff} \quad \forall j \geqslant 0.\ \sigma[j\ldots] \models \varphi
\end{array}
$$

$$
\begin{array}{llll}
\sigma & \models & \Box\Diamond\varphi & \text{iff} \quad \overset{\infty}{\exists}\ j.\ \sigma[j\ldots] \models \varphi \\
\sigma & \models & \Diamond\Box\varphi & \text{iff} \quad \overset{\infty}{\forall}\ j.\ \sigma[j\ldots] \models \varphi
\end{array}
$$

More Examples in Class