

# Assignment 3

## CSC373 Winter 2009

Due Date: April 9<sup>th</sup>, Noon, BA2220 Drop Box 7

### Problem 1

DPV 7.24

### Problem 2

Consider the Bin Packing problem: given positive integer  $B$ , and a list of integers  $A = [a_1, \dots, a_n]$  (where  $0 < a_i \leq B$ ), try to partition  $A$  into as few cells as possible such that for each cell of the partition, the sum of the elements in the cell is at most  $B$ .

The FirstFit algorithm takes each element of  $A$  in turn (note that we do NOT sort the objects by size) and places it into the first bin that can accommodate it.

For example, suppose  $B = 10$  and  $A = [2, 2, 7, 8, 3, 6, 3, 2, 6]$ . FirstFit would use 5 bins and fill them as follows: Bin(1) contains 2, 2, 3 and 3; Bin(2) contains 7 and 2; Bin(3) contains 8; Bin(4) contains 6; and Bin(5) contains 6.

Let  $S = \sum_{i=1}^n a_i$

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- Argue that the optimum number of bins required is at least  $\lceil S/B \rceil$ .
- Argue that the FirstFit algorithm leaves at most one bin less than half full.
- Prove that the number of bins used by FirstFit is never more than  $\lceil 2S/B \rceil$ .
- Prove an approximation ratio of 2 for the FirstFit algorithm. That is, prove that for any input  $A$  and integer  $B$ , FF uses at most twice the number of bins that an optimal algorithm would use.

### Problem 3

DPV 9.5

### Problem 4

DPV 7.16 (I like <http://vinci.inesc.pt/lp/>, once you get the hang of the syntax).