# Assignment 1
# CSC373 Winter 2010
# Due Date: Jan 28th, Noon, BA2220 Drop Box 8

## Problem 1

Given two sorted arrays, A[1…n] and B[1…k], we would like to find the ith largest element in the union of the two arrays.

a) Design an algorithm to solve the problem. Your description should be accurate and concise—pseudo-code is recommended. Justify why it is correct.

b) Describe the running time of your algorithm using a recurrence relation, and solve it. (NOTE: To get more than half the points for (a), your algorithm should run in O(log(n) + log(k)) time—otherwise you could get at most half the points for this problem)

## Problem 2

You are given the array A[1…n]. The elements of this array are either positive or negative integers. You are asked to find an interval of this array A[1< i…j < n] such that the sum of the elements in the interval $\sum_{k=i \dots j} A[k]$ is maximum.

a. How many intervals are there?

b. Design a simple, non-recursive algorithm that runs in time at most equal to the total number of intervals.

c. Design a recursive, divide an conquer algorithm for this problem that runs in time O(n log n). (Hint: Given the array and a position in the array, the optimal interval either contains that position, or is entirely contained within the left sub-array, or is entirely contained within the right sub-array)

## Problem 3

DPV 2.26

# Problem 4

Consider the following problem. The input consists of n skiers with heights $p_1, \ldots, p_n$, and n skies with heights $s_1, \ldots, s_n$. The problem is to assign each skier a ski to minimize the average difference between the height of a skier and his/her assigned ski. That is, if the ith skier is given the α(i)th ski, then you want to minimize:

$$\frac{1}{n}\sum_{i=1\ldots n}\left|p_i - s_{\alpha(i)}\right|$$

a) Consider the following greedy algorithm. Find the skier and ski whose height difference is minimized. Assign this skier this ski. Repeat the process until every skier has a ski. Prove of disprove that this algorithm is correct.

b) Consider the following greedy algorithm. Give the shortest skier the shortest ski, give the second shortest skier the second shortest ski, give the third shortest skier the third shortest ski, etc. Prove of disprove that this algorithm is correct.

# Problem 5

DPV 5.16

# Problem 6

Consider the problem of making change for n cents using as few coins as possible.

a) Describe a Greedy Algorithm to make change consisting of quarters, dimes, nickels and pennies. Prove that your algorithm yields an optimum solution.

b) Is the optimal solution always unique? (prove or disprove)

c) Give a set of coin denominations for which your Greedy Algorithm does not yield an optimum solution. Your set should include a penny, so that there is a solution for every value of n.