2534 Lecture 2: Utility Theory

- Tutorial on Bayesian Networks: Weds, Sept. 17, 5-6PM, PT266
- LECTURE ORDERING: Game Theory before MDPs? Or vice versa?

- Preference orderings
- Decision making under strict uncertainty
- Preference over lotteries and utility functions
- Useful concepts
  - Risk attitudes, certainty equivalents
  - Elicitation and stochastic dominance
- Paradoxes and behavioral decision theory
- Multi-attribute utility models
  - preferential and utility independence
  - additive and generalized addition models
Why preferences?

- Natural question: why not specify behavior with *goals*?
- Preferences: *coffee > OJ > tea*
  - Natural goal: coffee
    - but what if unavailable? requires a 30 minute wait? …
  - allows alternatives to be explored in face of costs, infeasibility,…
Preference Orderings

- Assume (finite) outcome set $X$ (states, products, etc.)

- Preference ordering $\succeq$ over $X$:
  - $y \succeq z$ interpreted as: “I (weakly) prefer $y$ to $z$”
  - $y > z$ iff $y \succeq z$ and $z \not\succeq y$ (strict preference)
  - $y \sim z$ iff $y \succeq z$ and $y \succeq z$ (indifference, incomparability?)

- Conditions: $\succeq$ must be:
  - (a) transitive: if $x \succeq y$ and $y \succeq z$ then $x \succeq z$
  - (b) connected (orderable): either $y \succeq z$ or $z \succeq y$
  - i.e., a total preorder
Preference Orderings

- Total preorder: seems natural, but conditions reasonable?
  - implies (iff) strict relation $>$ is asymmetric and neg. transitive*
    - *if a not better than b, b not better than c, then a not better than c
  - why connected? why transitive? (e.g., money pump)

- Are preference orderings enough?
  - decisions under certainty? under uncertainty?

- Exercise: what properties of $\succeq$, $>$ needed if you desire incomparability?
Revealed Preference

- Given a non-empty subset of $Y \subseteq X$, preferences “predict” choice: $c(Y) \in X$ should be a most preferred element
- More general choice function: select subset $c(Y) \subseteq Y$

- Given $\succ$, define $c(Y, \succ) = \{ y \in Y : \not\exists z \in Y \text{ s.t. } z \succ y \}$
  - i.e., the set of “top elements” of $\succ$ (works for partial orders too)
  - Exercise: show that $c(Y, \succ)$ must be non-empty
  - Exercise: show that if $y, z \in c(Y, \succ)$ then $y \sim z$

- CF $c$ is rationalizable iff exists $\succ$ s.t. for all $Y$, $c(Y) = c(Y, \succ)$
  - are all choice functions rationalizable? (give counterexample)
Weak Axiom of Revealed Preference

- Desirable properties of choice functions:
  - (AX1) If \( y \in Y, \ Y \subseteq Z, \) and \( y \in c(Z) \), then \( y \in c(Y) \)
  - (AX2) If \( Y \subseteq Z, \ y, z \in c(Y), \) and \( z \in c(Z) \), then \( y \in c(Z) \)

- **Thm:** (a) given prefs \( \succ \), \( c(\cdot, \succ) \) satisfies (AX1) and (AX2)
  (b) if \( c \) satisfies (AX1) and (AX2), then \( c = c(\cdot, \succ) \) for some \( \succ \)
  - Exercise: prove this

- Thus: a characterization of rationalizable choice functions

- Weak axiom of revealed preference:
  - (WARP) If \( y, z \in Y \cap Z, \ y \in c(Y), \ z \in c(Z) \), then \( y \in c(Z) \) and \( z \in c(Y) \)
  - Alternative characterization: \( c \) satisfies WARP iff (AX1) and (AX2)
Making Decisions: One-shot

Basic model of (one-shot) decisions:

- finite set of actions $A$, each leads to set of possible outcomes $X$
- given preference ordering $\succeq$, is decision obvious?

Deterministic actions: $f: A \rightarrow X$

- Let $f(A) = \{f(a) \in A\}$ be the set of possible outcomes, choose $a$ with most preferred outcome: $c(f(A))$
- preferences more useful than goals: what if $A$ is set of plans?

Is it always so straightforward?

$\quad x_1 \succ x_2 \succ x_3$: then choose $a_1$
Making Decisions: Uncertainty

- What if a given action has several possible outcomes
  - Nondeterministic actions: $f: A \rightarrow \mathcal{P}(X)$
  - Stochastic actions: $f: A \rightarrow \Delta(X)$
  - Initial state uncertainty (nondeterministic or stochastic)

\[
\begin{align*}
x_1 &> x_2 > x_3: \text{choose } a_1 \text{ or } a_2 ? \\
x_1 &> x_2 > x_3 > x_4: \text{choose } a_1 \text{ or } a_2 ?
\end{align*}
\]
Making Decisions: Uncertainty

Two solutions to this problem:

- Soln 1: Assign values to outcomes
  - decision making under strict uncertainty if nondeterministic
  - expected value/utility theory if stochastic
  - Question: where do values come from? what do they mean?

- Soln 2: Assign preferences to lotteries over outcomes
  - decision making under quantified uncertainty
Making Decisions: Strict Uncertainty

- Suppose you have no way to quantify uncertainty, but each outcome has some “value” to you
  - require the value function respect $\succeq$: $v(x) \geq v(y)$ iff $x \succeq y$

- Useful to specify a decision table
  - rows: *actions*; columns: *states of nature*; entries: *values*
  - unknown states of nature dictate outcomes, table has: $v(f(a, \Theta_i))$

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<thead>
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<th>$\Theta_2$</th>
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<th>$\Theta_k$</th>
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<td>$v_{n2}$</td>
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Strict Uncertainty: Decision Criteria

- **Maximin (Wald):** choose action with *best* worst outcome
  - $\max_a \min_\theta v(f(a, \theta))$
  - $a$ with max security level $s(a)$
  - very pessimistic

- **Maximax:** choose action with *best* best outcome
  - $\max_a \max_\theta v(f(a, \theta))$
  - $a$ with max optimism level $o(a)$

- **Hurwicz criterion:** set $\alpha \in (0, 1)$
  - $\max_a \alpha s(a) + (1- \alpha)o(a)$

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<tr>
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<td>0</td>
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<tr>
<td>$a_4$</td>
<td>1</td>
<td>3</td>
<td>0</td>
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- Maximin: $a_2$
- Maximax: $a_3$
- Hurwicz: *which decisions are possible?*
- *What if $a_3 = <0.5 \ 3 \ 2 \ 2>$?*
Minimax Regret (Savage)

- **Regret** of $a_i$ under outcome $\Theta_j$: $r_{ij} = \max \{v_{kj}\} - v_{ij}$
  - How sorry I’d be doing $a_i$ if I’d known $\Theta_j$ was coming
  - Why worry about worst outcome: beyond my control

- **Minimax regret**: choose $\arg\min_a \max_j r_{ij}$

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<td>1 / 0</td>
<td>2</td>
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<td>1 / 3</td>
<td>1 / 0</td>
<td>1 / 0</td>
<td>3</td>
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<tr>
<td>$a_3$</td>
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<td>4 / 0</td>
<td>0 / 1</td>
<td>0 / 1</td>
<td>2</td>
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<tr>
<td>$a_4$</td>
<td><strong>1 / 1</strong></td>
<td><strong>3 / 1</strong></td>
<td><strong>0 / 1</strong></td>
<td><strong>0 / 1</strong></td>
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*red values are regrets $r_{ij}$*
Qualitative Criteria: Reasonable?

- Criteria all make sense at some level, but not at others
  - indeed, all have “faults”

- Independence of irrelevant alternatives (IIA): adding an action to decision problem does not influence relative ranking of other actions

- Minimax regret violates IIA
  - $a_1$ lower MR than $a_2$ (no $a_3$)
  - $a_2$ lower MR than $a_1$ (with $a_3$)

- Classic impossibility result:
  - no qualitative decision criterion satisfies all of a set of intuitively reasonable principles (like IIA)

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<tbody>
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<td>9 / 0 / 0</td>
<td>3 / 1 / 5</td>
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<tr>
<td>$a_2$</td>
<td>2 / 4 / 4</td>
<td>9 / 0 / 0</td>
<td>4 / 0 / 4</td>
</tr>
<tr>
<td>$a_3$</td>
<td>0 / - / 6</td>
<td>0 / - / 9</td>
<td>8 / - / 0</td>
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*red: regrets $r_{ij}$ without $a_3$
*green: regrets $r_{ij}$ with $a_3$
Making Decisions: Probabilistic Uncertainty

- What if:
  - 2% chance no coffee made (30 min delay)? 10%? 20%? 95%?
  - robot has enough charge to check only one possibility
  - 5% chance of damage in coffee room, 1% at OJ vending mach.
Preference over Lotteries

- If uncertainty in action/choice outcomes, \(\geq\) not enough
- Each action is a “lottery” over outcomes

- A simple lottery over \(X\) has form:
  \[
  l = [ (p_1,x_1), (p_2,x_2), \ldots, (p_n,x_n) ]
  \]
  where \(p_i \geq 0\) and \(\sum p_i = 1\)
  - outcomes are just trivial lotteries (one outcome has prob 1)

- A compound lottery allows outcomes to be lotteries:
  \[
  [ (p_1,l_1), (p_2,l_2), \ldots, (p_n,l_n) ]
  \]
  - restrict to finite compounding
Constraints on Lotteries

- **Continuity:**
  \[ x_1 > x_2 > x_3 \] then \( \exists p \text{ s.t. } [(p,x_1), (1-p,x_3)] \sim x_2 \)

- **Substitutability:**
  \[ x_1 \sim x_2 \] then \([ (p,x_1), (1-p,x_3) ] \sim [(p,x_2), (1-p,x_3)] \)

- **Monotonicity:**
  \[ x_1 \succeq x_2 \text{ and } p \geq q \] then \([ (p,x_1), (1-p,x_2) ] \succeq [(q,x_1), (1-q,x_2)] \)

- **Reduction of Compound Lotteries (“no fun gambling”):**
  \[ [(p, [(q,x_1), (1-q,x_2)]), (1-p, [(q’,x_3), (1-q’,x_4)])] \]
  \[ \sim [(pq,x_1), (p-pq,x_2), (q’-pq’,x_3), ((1-p)(1-q’),x_4)] \]

- **Nontriviality:**
  \[ x_T > x_\perp \]
Implications of Properties on $\succeq$

- Since $\succeq$ is transitive, connected: representable by ordinal value function $V(x)$

- With constraints on lotteries: we can construct a utility function $U(l) \in \mathbb{R}$ s.t. $U(l_1) \geq U(l_2)$ iff $l_1 \succeq l_2$
  
  - where $U([ (p_1, x_1), \ldots, (p_n, x_n) ]) = \sum_i p_i U(x_i)$
  
  - famous result of Ramsey, von Neumann & Morgenstern, Savage

- Exercise: prove existence of such a utility function

- Exercise: given any $U$ over outcomes $X$, show that ordering $\succeq$ over lotteries induced by $U$ satisfies required properties of $\succeq$
Implications of Properties on

- Assume some collection of actions/choices at your disposal

- Knowing $U(x_i)$ for each outcome allows tradeoffs to be made over uncertain courses of action (lotteries)
  - simply compute expected utility of each course of action

- Principle of Maximum Expected Utility (MEU)
  - utility of choice is the expected utility of its outcome
  - appropriate choice is that with maximum expected utility
  - Why? Action (lottery) with highest EU is the action (lottery) that is most preferred in ordering $\succeq$ over lotteries!
Some Discussion Points

- Utility function existence: proof is straightforward
  - Hint: set \( U(x_T) = 1; U(x_\perp) = 0 \); find a \( p \) s.t. \( x \sim [(p,x_T), (1-p,x_\perp)] \)

- Utility function for \( > \) over lotteries is not unique:
  - any positive affine transformation of \( U \) induces same ordering \( > \)
  - normalization in range \([0,1]\) common

- Ordinal preferences “easy” to elicit (if \( X \) small)
  - cardinal utilities trickier for people: an “art form” in decision anal.

- Outcome space often factored: exponential size
  - requires techniques of multi-attribute utility theory (MAUT)

- Expected utility accounts for risk attitudes: inherent in preferences over lotteries
  - see utility of money (next)
Risk profiles and Utility of money

- What would you choose?
  - (a) $100,000 or (b) [(0.5, $200,000), (0.5, 0)]
  - what if (b) was $250K, $300K, $400K, $1M; p = 0.6, 0.7, 0.9, 0.999, ...
  - generally, $U(EMV(lottery)) > U(lottery)$  \( EMV = \text{expected monetary value} \)

- Utility of money is nonlinear: e.g., $U($100K) > 0.5U($200K) + 0.5U($0)$

- Certainty equivalent of $l$: $U(CE) = U(l)$; $CE = U^{-1}(EU(l))$

For many people, $CE \sim $40K
Note: 2\textsuperscript{nd} $100K “worth less” than 1\textsuperscript{st} $100K
Risk attitudes

- **Risk Premium**: $EMV(l) - CE(l)$
  - how much of EMV will I give up to remove risk of losing

- **Risk averse**:
  - decision maker has positive risk premium; $U(money)$ is concave

- **Risk neutral**:
  - decision maker has zero risk premium; $U(money)$ is linear

- **Risk seeking**:
  - decision maker has negative risk premium; $U(money)$ is convex

- Most people are risk averse
  - this explains insurance
  - often risk seeking in negative range
  - linear a good approx in small ranges
St. Peterburg Paradox

- How much would you pay to play this game?
  - A coin is tossed until it falls heads. If it occurs on the $N^{th}$ toss you get $2^N$

$$EMV = \sum_{n=1}^{\infty} \left( \frac{1}{2} \right)^n 2^n = \sum_{n=1}^{\infty} 1 = \infty$$

- Most people will pay about $2-$20

- Not a paradox per se... doesn’t contradict utility theory
A Game

- Situation 1: choose either
  - (1) $1M, Prob=1.00
  - (2) $5M, Prob=0.10; $1M, Prob=0.89; nothing, Prob=0.01
Another Game

- Situation 2: choose either
  - (3) $1M, Prob=0.11; nothing, Prob=0.89
  - (4) $5M, Prob=0.10; nothing, Prob=0.90
Allais’ Paradox

- **Situation 1:** choose either
  - (1) $1M, Prob=1.00
  - (2) $5M, Prob=0.10; $1M, Prob=0.89; nothing, Prob=0.01

- **Situation 2:** choose either
  - (3) $1M, Prob=0.11; nothing, Prob=0.89
  - (4) $5M, Prob=0.10; nothing, Prob=0.90

- **Most people:** (1) > (2) and (4) > (3)
  - e.g., in related setups: 65% (1) > (2); 25% (3) > (4)

- **Paradox:** no way to assign utilities to monetary outcomes that conforms to expected utility theory and the stated preferences (violates substitutability)
  - possible explanation: regret
Allais’ Paradox: The Paradox

- **Situation 1:** choose either
  - (1) $1M, Prob=1.00
    - *equiv:* ($1M 0.89; $1M 0.11)
  - (2) $5M, Prob=0.10; $1M, Prob=0.89; nothing, Prob=0.01
  - So if (1)>(2), by subst: $1M > ($5M 10/11; nothing 1/11)

- **Situation 2:** choose either
  - (3) $1M, Prob=0.11; nothing, Prob=0.89
  - (4) $5M, Prob=0.10; nothing, Prob=0.90
    - *equiv:* nothing 0.89; $5M 0.10; nothing 0.01
  - So if (4)>(3), by subst: ($5M 10/11; nothing 1/11) > $1M
...and the Fall 2014 survey says

- **Situation 1:**
  - (1)>(2): a (x%)
  - (2)>(1): b (y%)

- **Situation 2:**
  - (3)>(4): c (w%)
  - (4)>(3): d (z%)

- **The 2534 class of 2014 is ________________**
  - many people who take a class on decision theory tend to think in terms of expected monetary value (so 2534 surveys tend to be consistent than more standard empirical results; however, if there was real money on the line, my guess is the proportions would be somewhat more in line with experiments)
Ellsberg Paradox

- Urn with 30 red balls, 60 yellow or black balls; well mixed
- Situation 1: choose either
  - (1) $100 if you draw a red ball
  - (2) $100 if you draw a black ball
- Situation 2: choose either
  - (3) $100 if you draw a red or yellow ball
  - (4) $100 if you draw a black or yellow ball
- Most people: (1) > (2) and (4) > (3)
- Paradox: no way to assign utilities (all the same) and beliefs about yellow/black proportions that conforms to expected utility theory
  - possible explanation: ambiguity aversion
Utility Representations

- Utility function $u: X \rightarrow [0,1]$
  - decisions induce distribution over outcomes
  - or we simply choose an outcome (no uncertainty), but constraints on outcomes
- If $X$ is combinatorial, sequential, etc.
  - representing, eliciting $u$ difficult in explicit form
Product Configuration*

Luggage Capacity?
Two Door? Cost?
Engine Size?
Color? Options?
COACH*  

- POMDP for prompting Alzheimer’s patients  
  - solved using factored models, value-directed compression of belief space  

- Reward function (patient/caregiver preferences)  
  - indirect assessment (observation, policy critique)
Winner Determination in Combinatorial Auctions

- **Expressive bidding** in auctions becoming common
  - expressive languages allow: combinatorial bids, side-constraints, discount schedules, etc.
  - direct expression of utility/cost: economic efficiency

- **Advances in winner determination**
  - determine least-cost allocation of business to bidders
  - new optimization methods key to acceptance
  - applied to large-scale problems (e.g., sourcing)
Non-price Preferences

A and B for $12000.
C and D for $5000...

A for $10000.
B and D for $5000 if A;
B and D for $7000 if not A...

A, C to Fred.
B, D, G to Frank.
F, H, K to Joe...
Cost: $57,500.

That gives too much business to Joe!!
Non-price Preferences

- WD algorithms *minimize cost* alone
  - but preferences for *non-price attributes* play key role
  - Some typical attributes in sourcing:
    - *percentage volume business to specific supplier*
    - *average quality of product, delivery on time rating*
    - *geographical diversity of suppliers*
    - *number of winners (too few, too many), …*

- Clear utility function involved
  - difficult to articulate precise tradeoff weights
  - “What would you pay to reduce \( \%volume_{Joe} \) by 1%?”
Manual Scenario Navigation*

- Current practice: manual *scenario navigation*
  - impose constraints on winning allocation
    - *not a hard constraint!*
  - re-run winner determination
  - new allocation satisfying constraint: higher cost
  - assess tradeoff and repeat (often hundreds of times) until satisfied with some allocation

Here’s a new allocation with less business to Joe. **Cost is now: $62,000.**
Utility Representations

- Utility function $u: X \rightarrow [0,1]$
  - decisions induce distribution over outcomes
  - or we simply choose an outcome (no uncertainty), but constraints on outcomes
- If $X$ is combinatorial, sequential, etc.
  - representing, eliciting $u$ difficult in explicit form
- Some structural form usually assumed
  - so $u$ parameterized compactly (weight vector $w$)
  - e.g., linear/additive, generalized additive models

Representations for qualitative preferences, too
- e.g., CP-nets, TCP-nets, etc. [BBDHP03, BDS05]
Flat vs. Structured Utility Representation

- **Naïve representation:** vector of values
  - e.g., \( \text{car7:1.0, car15:0.92, car3:0.85, ..., car22:0.0} \)

- **Impractical for combinatorial domains**
  - e.g., can’t enumerate exponentially many cars, nor expect user to assess them all (choose among them)

- **Instead we try to exploit independence of user preferences and utility for different attributes**
  - the relative preference/utility of one attribute is independent of the value taken by (some) other attributes

- **Assume** \( X \subseteq \text{Dom}(X_1) \times \text{Dom}(X_2) \times \ldots \times \text{Dom}(X_n) \)
  - e.g., \( \text{car7: Color=red, Doors=2, Power=320hp, LuggageCap=0.52m}^3 \)
Preferential, Utility Independence

- **X** and **Y = V-X** are *preferentially independent* if:
  - $x_1y_1 \geq x_2y_1$ iff $x_1y_2 \geq x_2y_2$ (for all $x_1, x_2, y_1, y_2$)
  - e.g., *Color: red* > *blue* regardless of value of *Doors, Power, LugCap*
  - conditional P.I. given set $Z$: definition is straightforward

- **X** and **Y = V-X** are *utility independent* if:
  - $l_1(Xy_1) \geq l_2(Xy_1)$ iff $l_1(Xy_2) \geq l_2(Xy_2)$ (for all $y_1, y_2$, all distr. $l_1, l_2$)
  - e.g., preference for *lottery*(*Red, Green, Blue*) does not vary with value of *Doors, Power, LugCap*
    - implies existence of a “utility” function over local (sub)outcomes
  - conditional U.I. given set $Z$: definition is straightforward
Question

- Is each attribute PI of others in preference relation 1? 2?

- Does UI imply PI? Does PI imply UI?
Additive Utility Functions

- **Additive representations** commonly used [KR76]
  - breaks exponential dependence on number of attributes
  - use sum of *local utility functions* $u_i$ over attributes
  - or equivalently *local value functions* $v_i$ plus scaling factors $\lambda_i$

  $$u(x) = \sum_{i=1}^{n} u_i(x_i) = \sum_{i=1}^{n} \lambda_i v_i(x_i).$$

  - e.g., $U(\text{Car}) = 0.3 \ v_1(\text{Color}) + 0.2 \ v_2(\text{Doors}) + 0.5 \ v_3(\text{Power})$
    and $v_1(\text{Color}) : \text{cherryred}:1.0, \text{metallicblue}:0.7, \ldots, \text{grey}:0.0$

- This will make elicitation much easier (more on this next time)
- It *can* also make optimization more practical (more next time)
Additive Utility Functions

- An additive representation of $u$ exists iff decision maker is indifferent between any two lotteries where the marginals over each attribute are identical.
  - $l_1(X) \sim l_2(X)$ whenever $l_1(X_i) = l_2(X_i)$ for all $X_i$

$$u(x) = \sum_{i=1}^{n} u_i(x_i) = \sum_{i=1}^{n} \lambda_i v_i(x_i)$$
Generalized Additive Utility

- **Generalized additive models** more flexible
  - **interdependent value additivity** [Fishburn67], GAI [BG95]
  - assume (overlapping) set of \( m \) subsets of vars \( X[j] \)
  - use sum of *local utility functions* \( u_j \) over attributes

\[
 u(x) = \sum_{j=1}^{m} u_j(x_j)
\]

- e.g., \( U(\text{Car}) = 0.3 \, v_1(\text{Color}, \text{Doors}) + 0.7 \, v_2(\text{Doors}, \text{Power}) \) with
  
  \[
  v_1(\text{Color}, \text{Door}) : \quad \text{blue, sedan:} 1.0; \quad \text{blue, coupe:} 0.7; \quad \text{blue, hatch:} 0.1,
  \quad \text{red, sedan:} 0.8; \quad \text{red, coupe:} 0.9; \quad \text{red, hatch:} 0.0
  \]

- This will make elicitation much easier (more on this next time)
- It *can* also make optimization more practical (more next time)
GAI Utility Functions

- An GAI representation of $u$ exists iff decision maker is indifferent between any two lotteries where the marginals over each factor are identical
  - $l_1(X) \sim l_2(X)$ whenever $l_1(X[i]) = l_2(X[i])$ for all $i$

\[
 u(x) = \sum_{j=1}^{m} u_j(x_j)
\]
Further Background Reading