University of Toronto
csc148F—Introduction to Computer Science, Fall 1997

Second Test — Horton's evening section

Solutions!

Duration: 50 minutes

Aids allowed: None

Make sure that your examination booklet has 5 pages (including this one). Write your answers in the spaces provided. Write legibly.

Family Name: _____

First Name: _____

Student # : _____

Tutor (circle one): Cyrus Hsia Catherine Stinson Shingo Takagi Jamie Hale

1. _____ / 10
2. _____ / 9
3. _____ / 9
4. _____ / 8

Total _____ / 36

Question 1

[10 marks in total]

For integers $n \geq 0$, let $S(n)$ be some statement. For the purpose of this question, it does not matter what this statement is.

For each proof outline given below, state whether or not it is a valid approach to proving that $S(n)$ is true for all integers $n \geq 0$. In other words, if you filled in the details, would you be able to conclude that $S(n)$ is true for all integers $n \geq 0$.

Circle YES or NO. If you circle NO, you must give your reason in order to earn any marks.

(a) Is this a valid approach to doing the proof? YES [NO], and reason:

Proof:

BASE CASE: Prove $S(0)$ is true. `B.C. doesn't connect to induction part`

Let $k > 0$ be an arbitrary integer.

INDUCTION HYPOTHESIS: Assume $S(k)$ is true.

INDUCTION STEP: Prove $S(k+1)$ is true.

(b) Is this a valid approach to doing the proof? [YES] NO, and reason:

Proof:

BASE CASE: Prove $S(0)$ is true.

Let $k \geq 1$ be an arbitrary integer.

INDUCTION HYPOTHESIS: Assume $S(k-1)$ is true.

INDUCTION STEP: Prove $S(k)$ is true.

(c) Is this a valid approach to doing the proof? [YES] NO, and reason:

Proof:

BASE CASE: Prove $S(0)$ is true and $S(1)$ is true.

Let $k \geq 0$ be an arbitrary integer.

INDUCTION HYPOTHESIS: Assume $S(k)$ is true.

INDUCTION STEP: Prove $S(k+2)$ is true.

(d) Is this a valid approach to doing the proof? YES [NO], and reason:

Proof:

BASE CASE: Prove $S(0)$ is true. `Induction step assumes the whole theorem!`

INDUCTION HYP: Assume $S(k)$ is true for all integers $k \geq 0$.

INDUCTION STEP: Prove $S(k+1)$ is true.

(e) Is this a valid approach to doing the proof? [YES] NO, and reason:

Proof:

BASE CASE: Prove $S(0)$ is true.

Let $k \geq 0$ be an arbitrary integer.

INDUCTION HYP: Assume $S(j)$ is true for all integers j where $0 \leq j \leq k$.

INDUCTION STEP: Prove $S(k+1)$ is true.

Question 2

[9 marks in total]

Consider the following Java method:

```
public static int smallest (int[] a, int size) {
    int small = 1000;
    int i = 0;

    while (i != size) {
       if (a[i] < small)
             small = a[i];
       i++;
    }
    return small;
}
```

There are several good solutions to this question. Below two are shown. Their answers cannot be mixed and matched though!

(a) [5 marks]
Give appropriate preconditions and postconditions for this method.

Preconditions:

Solution (1):
1 <= size <= a.length
at least one value in a[0..size-1] is less than 1000

Solution (2):
0 <= size <= a.length

Postconditions:

Solution (1)
the value returned is the smallest value in a[0..size-1] inclusive.

Solution(2):
if there is any value in a[0..size-1] inclusive that is < 1000
the smallest of the values in a[0..size-1] inclusive is returned;
otherwise, 1000 is returned.

(b) [4 marks]
Fill in each blank with a comparison operator (that is, one of: $=, \neq, <, \leq, >, \geq$) to make each of the following conditions a loop invariant.

i $\boxed{\leq}$ size.

If small $<$ 1000 then small $\leq$ a[j] for *all* integers j such that 0 $\boxed{\leq}$ j $\boxed{<}$ i.

If small $<$ 1000 then small $=$ a[j] for *some* integer j such that 0 $\boxed{\leq}$ j $\boxed{<}$ i.

Question 3
[9 marks in total]

Consider the following Java method:

```
private static int power (int x, int n) {
    System.out.println ("x=" + x + " n=" + n);
    if (n == 0)
        return 1;
    else {
        int k = power (x*x, n/2); // The '/' does integer division.
        System.out.println ("k=" + k);
        if ( (n%2) == 0 )         // Checks if n is even.
             return k;
        else
             return x * k;
    }
 }
```

(a) [7 marks]
What output would result from the following statement:

```
System.out.println (power(2,6) + " is the final answer.");
```

THE OUTPUT IS:

```
x=2 n=6
x=4 n=3
x=16 n=1
x=256 n=0
k=1
k=16
k=64
64 is the final answer.
```

(b) [2 marks]
If you were to prove the `power` method correct, what would you do induction on?

One would do induction on n.

Question 4

[8 marks in total]

The `StrNode` class, given here, can be used for a linked list of strings.

```
class Node {
    public String s;
    public Node next;

    // constructor: details omitted
}
```

Write a method called `longest`, which, given a linked list of strings, will return the length of the longest string in the list. If the list is empty, it should return 0. Your solution **must** be recursive.

Recall that `s.length()` gives the length of a string `s`.

```
public static int longest (Node first) {


ANSWER:


     if (first==null)
         return 0;
     else
         return Math.max(first.s.length(), longest(first.next));

}
```