University of Toronto
csc148F—Introduction to Computer Science, Fall 1997

Second Test — Horton's evening section

Duration: 50 minutes

Aids allowed: None

Make sure that your examination booklet has 5 pages (including this one). Write your answers in the spaces provided. Write legibly.

Family Name: ____________________

First Name: ____________________

Student # : ____________________

Tutor (circle one): Cyrus Hsia Catherine Stinson Shingo Takagi Jamie Hale

1.	________	/ 10
2.	________	/ 9
3.	________	/ 9
4.	________	/ 8
Total	________	/ 36

Question 1

[10 marks in total]

For integers $n \geq 0$, let $S(n)$ be some statement. For the purpose of this question, it does not matter what this statement is.

For each proof outline given below, state whether or not it is a valid approach to proving that $S(n)$ is true for all integers $n \geq 0$. In other words, if you filled in the details, would you be able to conclude that $S(n)$ is true for all integers $n \geq 0$.

Circle YES or NO. If you circle NO, you must give your reason in order to earn any marks.

(a) Is this a valid approach to doing the proof? YES NO, and reason:

Proof:
BASE CASE: Prove $S(0)$ is true.
Let $k > 0$ be an arbitrary integer.
INDUCTION HYPOTHESIS: Assume $S(k)$ is true.
INDUCTION STEP: Prove $S(k+1)$ is true.

(b) Is this a valid approach to doing the proof? YES NO, and reason:

Proof:
BASE CASE: Prove $S(0)$ is true.
Let $k \geq 1$ be an arbitrary integer.
INDUCTION HYPOTHESIS: Assume $S(k-1)$ is true.
INDUCTION STEP: Prove $S(k)$ is true.

(c) Is this a valid approach to doing the proof? YES NO, and reason:

Proof:
BASE CASE: Prove $S(0)$ is true and $S(1)$ is true.
Let $k \geq 0$ be an arbitrary integer.
INDUCTION HYPOTHESIS: Assume $S(k)$ is true.
INDUCTION STEP: Prove $S(k+2)$ is true.

(d) Is this a valid approach to doing the proof? YES NO, and reason:

Proof:
BASE CASE: Prove $S(0)$ is true.
INDUCTION HYP: Assume $S(k)$ is true for all integers $k \geq 0$.
INDUCTION STEP: Prove $S(k+1)$ is true.

(e) Is this a valid approach to doing the proof? YES NO, and reason:

Proof:
BASE CASE: Prove $S(0)$ is true.
Let $k \geq 0$ be an arbitrary integer.
INDUCTION HYP: Assume $S(j)$ is true for all integers j where $0 \leq j \leq k$.
INDUCTION STEP: Prove $S(k+1)$ is true.

Question 2

[9 marks in total]

Consider the following Java method:

```
public static int smallest (int[] a, int size) {
    int small = 1000;
    int i = 0;

    while (i != size) {
       if (a[i] < small)
             small = a[i];
       i++;
    }
    return small;
}
```

(a) [5 marks]
Give appropriate preconditions and postconditions for this method.
Preconditions:

Postconditions:

(b) [4 marks]
Fill in each blank with a comparison operator (that is, one of: $=, \neq, <, \leq, >, \geq$) to make each of the following conditions a loop invariant.

`i` ______ `size`.

If `small` < 1000 then `small` $\leq$ `a[j]` for *all* integers `j` such that 0 ______ `j` ______ `i`.

If `small` < 1000 then `small` $=$ `a[j]` for *some* integer `j` such that 0 ______ `j` ______ `i`.

Question 3

[9 marks in total]

Consider the following Java method:

```
private static int power (int x, int n) {
    System.out.println ("x=" + x + " n=" + n);
    if (n == 0)
        return 1;
    else {
        int k = power (x*x, n/2); // The '/' does integer division.
        System.out.println ("k=" + k);
        if ( (n%2) == 0 )         // Checks if n is even.
             return k;
        else
             return x * k;
    }
 }
```

(a) [7 marks]
What output would result from the following statement:

```
System.out.println (power(2,6) + " is the final answer.");
```

(b) [2 marks]
If you were to prove the `power` method correct, what would you do induction on?

Question 4
[8 marks in total]

The StrNode class, given here, can be used for a linked list of strings.

```
class Node {
    public String s;
    public Node next;

    // constructor: details omitted
}
```

Write a method called longest, which, given a linked list of strings, will return the length of the longest string in the list. If the list is empty, it should return 0. Your solution **must** be recursive.

Recall that s.length() gives the length of a string s.

```
public static int longest (Node first) {
```