Mathematics Preparedness Summer Learning Institute Summer 2014 Assignment 1

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Due Thursday August 14, in class

Total possible marks: 120 marks

IMPORTANT: In this and in future assignments, ten percents (10%) of the total marks are allocated to the quality of the presentation your solutions.

Make sure to have your assignment stapled and to write your name and student number on the first page.

Assignment Questions

1. (20 marks: 5 marks for each part)

Let

$$A = \{x | x \ge -3\}, \qquad B = \{x | x \le 5\}$$
$$C = \{x | -2 \le x \le 6\}.$$

Write the following sets in the interval notation and graph on the real line:

 $B\cup C, \quad B\cap C, \quad A\cap B, \quad A\cap C.$

2. (10 marks: 5 marks for each part)

Simplify the following expressions.

(a)
$$\frac{\frac{1}{a+h} - \frac{1}{a}}{h}$$

(b) $\frac{x^{-2} - y^{-2}}{x^{-1} + y^{-1}}$

3. (20 marks: 5 marks for each part)

Factor each expressions.

(a)
$$3x^{3/2} - 9x^{1/2} + 6x^{-1/2}$$

(b) $8x^3 - 125y^6$
(c) $x^3 + 4x^2 + x + 4$
(d) $(x^2 + 1)^2 - 7(x^2 + 1) + 10$

4. (10 marks: 5 marks for each part)

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Rationalize the numerator or the denominator of the following expressions.

(a)
$$\frac{2(x-y)}{\sqrt{x}-\sqrt{y}}$$

(b) $\sqrt{x+1}-\sqrt{x}$

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5. (5 marks)

Perform the operation and simplify

$$(3x^2 - 4x + 3) \div (3x + 2).$$

6. (20 marks: 5 marks for each part)

Solve the following equations for x:

(a) $a^{2}x + (a-1) = (a+1)x$ (b) $x^{2} + 2x - 5 = 0$ (c) $\frac{1}{x-1} + \frac{1}{x+2} = \frac{5}{4}$ (d) $x^{4/3} - 5x^{2/3} + 6 = 0$

7. (10 marks)

A machine company has an incentive plan for its salespeople. For each machine that a salesperson sells, the commission is \$40. The commission for *every* machine sold will increase by \$0.04 for each machine sold over 600. For example, the commission on each of 602 machines sold is \$40.08. How many machines must a salesperson sell in order to earn \$30,000?

8. (10 marks)

A bottle contains 750 ml of fruit punch with a concentration of 50% pure fruit juice. Jill drinks 100 ml then refills the bottle with an equal amount of a cheaper branch of punch. If the concentration of the juice in the bottle is now reduced to 48%, what was the concentration in the punch that Jill added?

9. (15 marks)

Prove that if 0 < a < b, then

$$a < \sqrt{ab} < \frac{a+b}{2} < b.$$

Practice Questions - Do not hand in Similar questions may appear on your final assessment

10. (0 marks)

Prove the following:

(a)
$$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$$

(b) $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$
(c) $x^n - y^n = (x - y)(x^{n-1} + x^{n-2}y + \ldots + xy^{n-2} + y^{n-1})$

11. (0 marks)

Prove that if $0 \le x < y$ then

$$x^n < y^n, \quad n = 1, 2, 3, \dots$$
 (1)

Show that (1) may not hold true if the condition $0 \le x < y$ is not satisfied.