
Mathematics Preparedness Summer Learning Institute
Summer 2014
Assignment 2

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Due Tuesday August 19, in class	Total possible marks: 110 marks
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IMPORTANT: In this and in future assignments, ten percents (10%) of the total marks are allocated to the quality of the presentation of your solutions.

Make sure to have your assignment stapled and to write your name and student number on the first page.

Assignment Questions

1. (10 marks: 5 marks for each part)

Solve the following inequalities

(a) $1 + \frac{2}{x+1} \leq \frac{2}{x}$

(b) $\left| \frac{x-2}{3} \right| < 2$

2. (10 marks)

Show that $A(1,3)$, $B(5,3)$, $C(1,-3)$, and $D(5,-3)$ are vertices of a rectangle. Find the area of this rectangle.

3. (10 marks: 5 marks for each part)

Identify and draw the following graphs:

(a) $x^2 + y^2 - 4x + 10y + 13 = 0$

(b) $2x^2 + y^2 - 12x + 2y + 19 = 0$

4. (10 marks) Let

$$f(x) = \begin{cases} x^2 + 2x & \text{if } x \leq -1 \\ x & \text{if } -1 < x \leq 1 \\ -1 & \text{if } x > 1 \end{cases}$$

Find $f(-4) + f(-3/2) + f(-1) + f(0) + f(25)$.

5. (15 marks: 5 marks for each part)

Find the domain of the following functions

(a) $f(x) = \begin{cases} 4 & \text{if } x = 3 \\ x^2 & \text{if } 1 \leq x < 3 \end{cases}$

(b) $\frac{x^4}{x^2 + x - 6}$

(c) $\frac{3}{\sqrt{x-4}}$

6. (10 marks: 5 marks for each part)

Determine whether f is even, odd or neither

(a) $f(x) = x + \frac{1}{x}$.

(b) $f(x) = 1 - \sqrt[3]{x}$.

7. (10 marks)

Given the function h

$$h(x) = \frac{x^2}{x^2 + 4}.$$

Find functions f , g and k such that $h = f \circ g \circ k$?

8. (15 marks: 5 marks for each part)

Given function $f(x) = \sqrt{x-2}$.

(a) Sketch the graph of $f(x)$.

(b) Use the graph of f to sketch the graph of f^{-1} .

(c) Find an equation for f^{-1} .

9. (10 marks: 5 marks for each part)

Sketch the graphs of

(a) $f(x) = 4x - x^2$ using the graph of $g(x) = x^2$.

(b) $h(x) = |4x - x^2|$ using the graph of $f(x) = 4x - x^2$.

10. (10 marks) Show that $\sqrt{3}$ is irrational.

Practice Questions - Do not hand in
Similar questions may appear on your final assessment

11. (0 marks)

Prove that any function f with domain \mathcal{R} can be written as

$$f = g + h,$$

where g is an even function and h is an odd function. Is this way of writing is unique? Mathematically justify your answer.

12. (0 marks)

Describe the graph of g in terms of the graph of f if

(a) $g(x) = f(|x|)$.

(b) $g(x) = f(1/x)$.

(c) $g(x) = \max(f(x), 0) + \min(f(x), 0)$.