
Mathematics Preparedness Summer Learning Institute
Summer 2014
Assignment 3

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Due Thursday August 21 by 5pm

Total possible marks: 100 marks

IMPORTANT: In this and in future assignments, ten percents (10%) of the total marks are allocated to the quality of the presentation of your solutions.

Make sure to have your assignment stapled and to write your name and student number on the first page.

Assignment Questions

1. (10 marks: 5 marks for each part)

Determine the domain of the following functions:

(a) $f(x) = \frac{1}{\sqrt{x^3 e^x - 7x e^x - 6e^x}}$

(b) $f(x) = \sqrt{x+5} - \log_3(3-x)$

2. (15 marks: 5 marks for each part)

Solve the following equations:

(a) $\log_2 3 + \log_2 x = \log_2 5 + \log_2(x-2)$

(b) $\log_2(x^2 - x - 2) = 2$

(c) $3^{4x} = 9^{x+1}$

3. (15 marks: 5 marks for each part)

The number of milligrams of a radioactive substance present after t years is given by

$$Q(t) = 100e^{-0.035t}.$$

- (a) What is the relative rate of reduction of the substance? Express your answer as a percentage.
(b) How many milligrams are present after 0 years?
(c) After how many years will there be 20 milligrams present?

4. (20 marks: 5 marks for each part)

Find the value of the given limit. If the limit does not exist, explain why.

(a) $\lim_{x \rightarrow 2} \frac{x^4 + x^3 - 24}{x^2 - 4}$

(b) $\lim_{x \rightarrow 6} \frac{\sqrt{x-2} - 2}{x-6}$

(c) $\lim_{x \rightarrow 2} f(x)$ where

$$f(x) = \begin{cases} \frac{x^3 - 8}{x - 2} & x < 2, \\ 9x^2 & x > 2. \end{cases}$$

(d) $\lim_{x \rightarrow 1^+} \frac{x^2 - 1}{|x - 1|}$ and $\lim_{x \rightarrow 1^-} \frac{x^2 - 1}{|x - 1|}$

5. (15 marks)

Let

$$f(x) = \begin{cases} \sqrt{2 - x} & x < 2, \\ x^3 + k(x + 1) & x \geq 2 \end{cases}$$

Find real number k such that $f(x)$ is continuous every where.

6. (15 marks: 10 marks for part (a))

Suppose that the long-distance rate for a phone call from Toronto to Ottawa is \$0.1 for the first minute and \$0.06 for each additional minutes or fraction thereof. Let $y = f(t)$ be a function that indicates the total charge for a call of t minutes with $0 < t \leq 4\frac{1}{2}$.

(a) Sketch the graph of $f(t)$ for $0 < t \leq 4\frac{1}{2}$.

(b) Use the graph of $f(t)$ to determine the values of t where $0 < t \leq 4\frac{1}{2}$, at which discontinuities occur.

7. (10 marks)

Show that

$$\lim_{x \rightarrow 0} x^2 \sin \frac{1}{x} = 0.$$