University of Toronto Scarborough

Mathematics Preparedness Summer Learning Institute
Summer 2014
Final Assessment

Student Last Name: _____

Student First Name: _____

Student ID Number: _____

Course Title:	*Mathematics Preparedness Summer Learning Institute*
Instructor:	*Dr. Duy-Minh Dang*

Date of Exam:	*Monday, August 25, 2014*
Location:	*IC 130*
Time Period:	Start time: *1:30 PM* End time: *3:30 PM*
Duration of Exam:	*2 hours*
Number of Exam Pages: (including this cover sheet)	4
Materials Allowed:	*A non-graphing calculator.*

Please answer ALL questions.
Write your solutions in the Examination book.
Write clearly and legibly. Unreadable answers will NOT be marked.
The marks total to 100 for all required questions and 10 points for the bonus question.
Five percents (5%) of the total marks are allocated to the quality of the presentation of your solutions.

Good luck!

1. (20 marks: 5 marks for each part)

For each of the following functions, first find its domain (1 marks) and then find its roots by any method (3 marks). (Note: roots of function $f(x)$ are solutions of the equation $f(x) = 0$.)

(a) $\log_2(x) - 5 + \log_2(x + 4)$

(b) $\dfrac{2}{x^2 - 1} - \dfrac{1}{x(x - 1)} - \dfrac{2}{x^2}$

(c) $2x - 1 + \sqrt{2 - x}$

(d) $|2x + 3| - x$

(e) $2\cos^2(x) + \sin(x) - 1$

2. (9 marks: 3 marks for each part)

For each of the following functions, determine whether the function is even, odd or neither (2 marks). Comment on the symmetry of the function with respect to the y-axis or the origin (1 marks).

(a) $f(x) = (x^4 + x^2 - 1)\cos(x)$

(b) $h(x) = x^3 + 3x - \mathrm{e}^x$

(c) $g(x) = |x|\sin(x)$

3. (10 marks: 5 marks for each part)

Solve the following inequalities and express the solution using interval notation.

(a) $x < \dfrac{2}{x - 1}$

(b) $\left|\dfrac{x + 1}{2} - \dfrac{x - 1}{3}\right| > 1$

4. (10 marks: 5 marks for each part)

(a) Given $f(x) = \sqrt{x - 3}$, $g(x) = x^2$ and $h(x) = x^3 + 2$. Find $f \circ g \circ h$. Do not simplify your answer.

(b) Given $h(x) = 1 - 3^{x^2}$. Find function f, g and k such that $h = f \circ g \circ k$.

5. (12 marks)

In a certain country, the tax on incomes less than or equal to $\$20,000$ is 10%. For incomes more than $\$20,000$, the tax is $\$2000$ plus 20% of the amount over $\$20,000$.

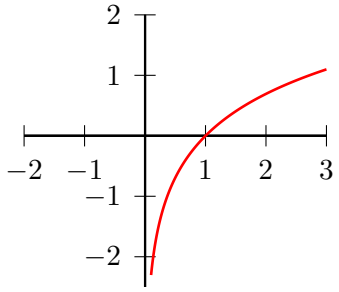
(a) (5 marks) Find a function f that gives the income tax on an income x. Express f as a case-defined function.

(b) (5 marks) Find f^{-1}. What does f^{-1} represent?

(c) (2 marks) How much income would require paying a tax of $\$10,000$?

6. (8 marks)

Given the graph of function $g(x) = \ln(x)$.



Sketch the graphs of

(a) (5 marks) $f(x) = 2\ln(x-3)$ using the graph of $g(x) = \ln(x)$.

(b) (3 marks) $h(x) = 2|\ln(x-3)|$ using the graph of $f(x) = 2\ln(x-3)$.

To get full marks, you must indicate the transformations used.

7. (15 marks: 5 marks for each part)

Find the value of the given limit. If the limit does not exist, explain why.

(a) $\displaystyle\lim_{x\to 3} \frac{x^4 + x^3 - 108}{x^2 - 9}$

(b) $\displaystyle\lim_{x\to 2} \frac{\sqrt{x-1} - 1}{x - 2}$

(c) $\displaystyle\lim_{x\to 0^+} \frac{x}{|x|} + \lim_{x\to 0^-} \frac{x}{|x|}$

8. (8 marks)

Let

$$f(x) = \begin{cases} \sqrt{6-x} & x < 6, \\ x^2 + kx + 1 & x \geq 6. \end{cases}$$

Find real number k such that $f(x)$ is continuous every where.

9. (8 marks)

Show that

$$\lim_{x\to 0} \left[x \cos\left(\frac{1}{x}\right) \right] = 0.$$

10. (BONUS 10 marks: 5 marks for each part)

(a) Show that the sum of two irrational numbers is not necessarily irrational. (Hint: give an example of two numbers $a, b \in \mathcal{I}$ such that $(a + b) \in \mathcal{Q}$).

(b) Consider the statement: "The sum of a rational number and an irrational number must be rational". Is it true or false? Mathematically justify your answer. (Hint: You can start by first assuming that this statement is true, i.e. for $a \in \mathcal{I}$ and $b \in \mathcal{Q}$, $(a + b) \in \mathcal{Q}$, and then considering $(a + b) - b$.)