

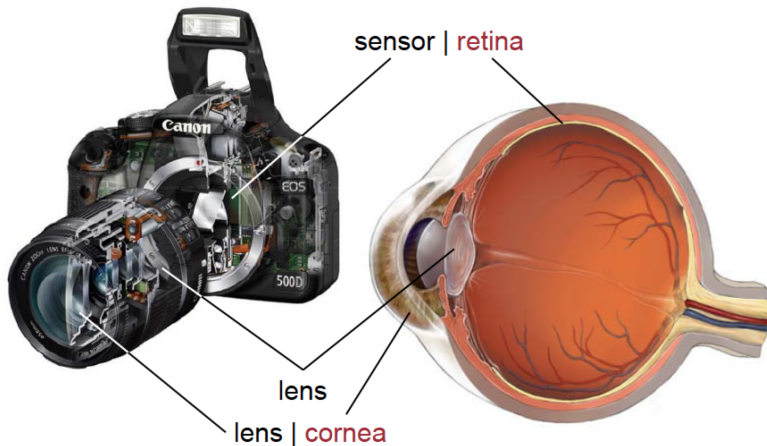
Camera Models

- If you are interested, this book has it all:

A. Zisserman and R. Hartley
Multiview Geometry
Cambridge University Press, 2003

Camera

- Camera is structurally similar to the eye



[Source: L.W. Kheng]

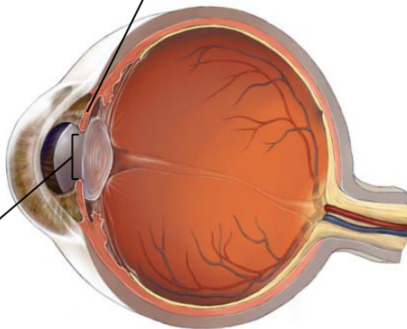
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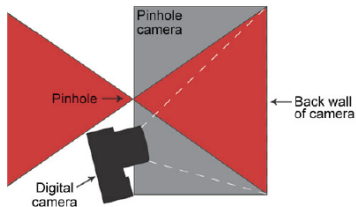
aperture | pupil

aperture
plates | iris



Camera

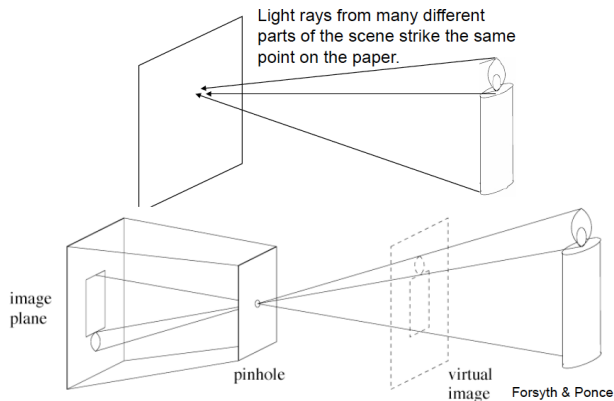
- Remember the pinhole camera from Lecture 2?



[Source: A. Torralba]

Camera

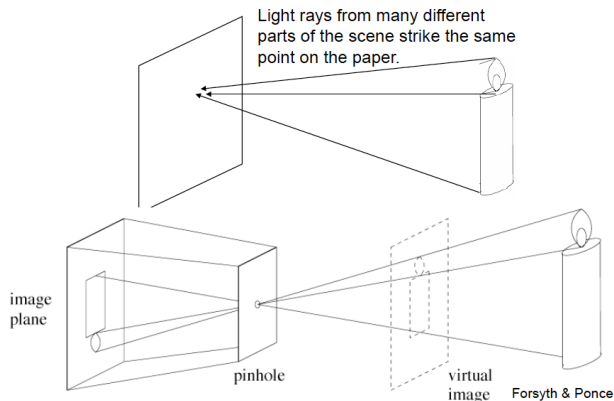
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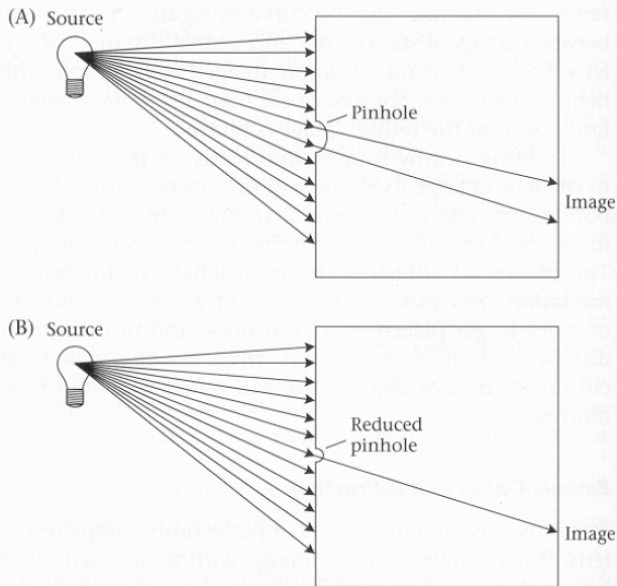
- Remember the pinhole camera from Lecture 2?
- Size of the pinhole is called **aperture**



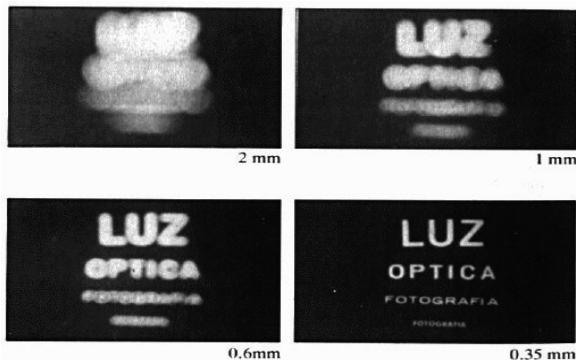
[Source: A. Torralba]

Pinhole Camera

[Source: A. Torralba]



Shrinking the Aperture

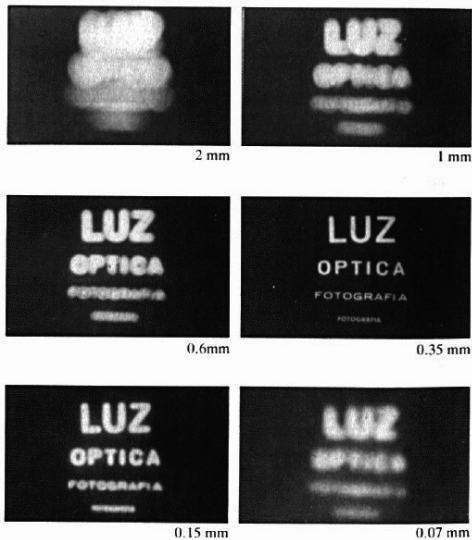


Why not make the aperture as small as possible?

- Less light gets through
- Diffraction effects...

[Source: N. Snavely]

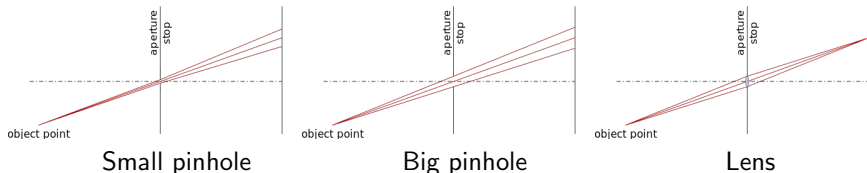
Shrinking the Aperture



[Source: N. Snavely]

Adding a Lens

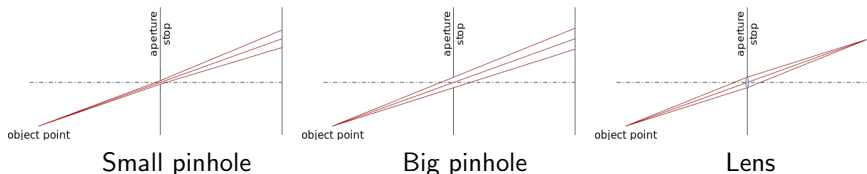
[Pic from Wikipedia]



- A lens focuses light onto the film
- There is a specific distance at which objects are **in focus**

Adding a Lens

[Pic from Wikipedia]

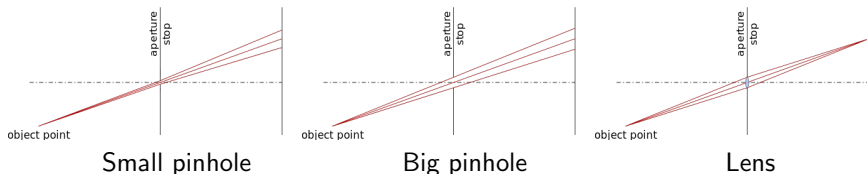


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Some “Cameras” Have Bigger Lenses than Others



http://www.use.com/images/s_2/thick_glasses_13b6941623c255ff400a_1.jpg?

Imaging

- Images are 2D projections of real world scene
- Images capture two kinds of information:
 - Geometric: positions, points, lines, curves, etc.
 - Photometric: intensity, color
- Complex 3D-2D relationships
- Camera models approximate these relationships

[Source: L.W. Kheng]

Projection



[Source: N. Snavely]

Projection



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3D to 2D Projection

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- We can do this using a linear 3D to 2D projection matrix

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[source: R. Urtasun]

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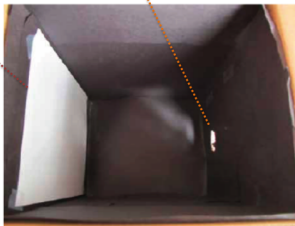
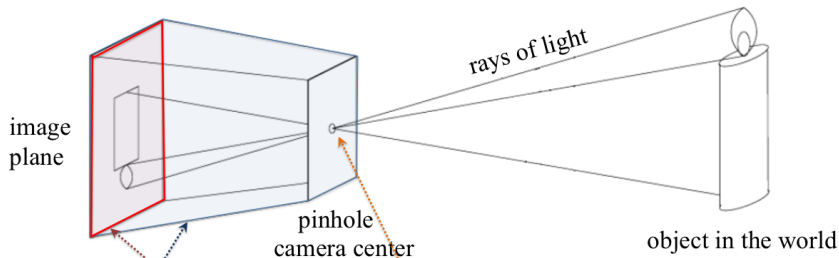
- How are 3D primitives projected onto the image plane?
- We can do this using a linear 3D to 2D projection matrix
- Different types, most common:
 - Perspective projection
 - Orthographic projection
 - Scaled orthographic projection
 - Paraperspective projection

[source: R. Urtasun]

Modeling Projection

[Pics from: A. Torralba, Forsyth & Ponce]

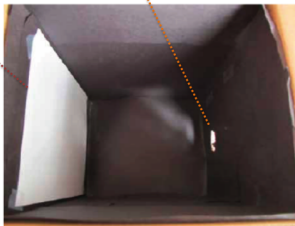
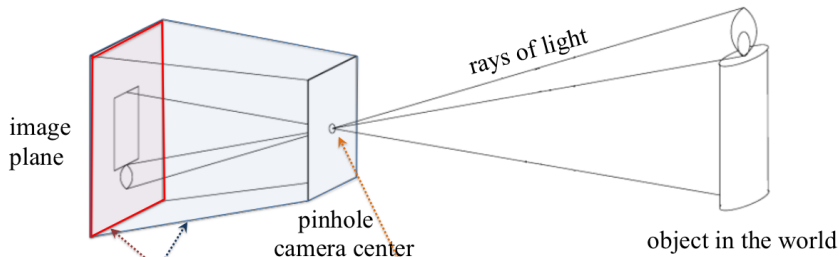
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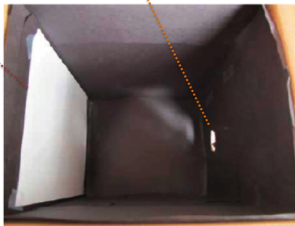
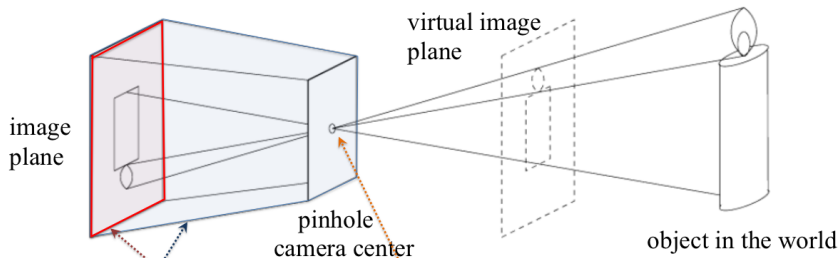


what the image plane sees

Modeling Projection

[Pics from: A. Torralba, Forsyth & Ponce]

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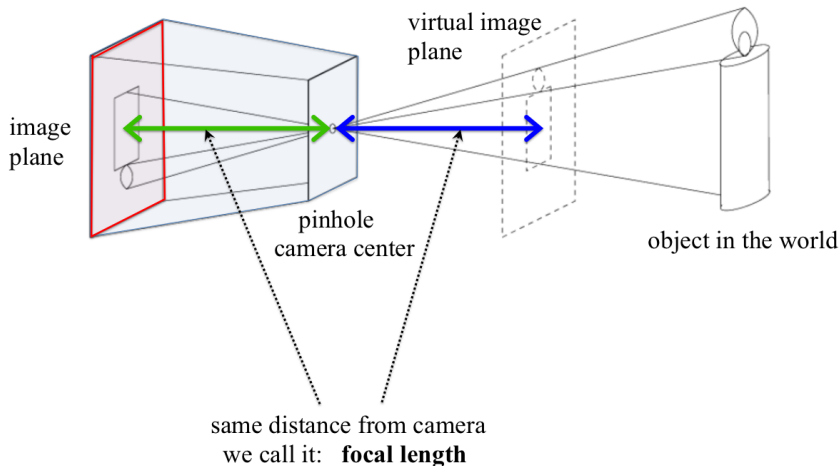


virtual image

Modeling Projection

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- We will use the pinhole model as an approximation



Focal Length

- Can be thought of as **zoom**
- Related to the **field of view**



24mm



50mm



200mm



800mm



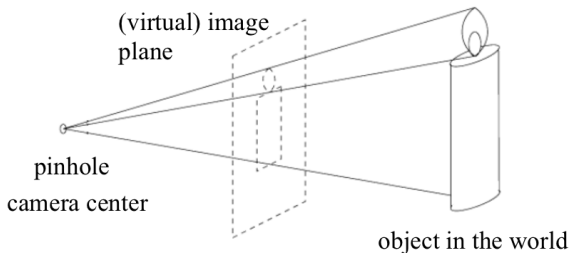
Figure: Image from N. Snavely

[Source: N. Snavely, slide credit: R. Urtasun]

Modeling Projection

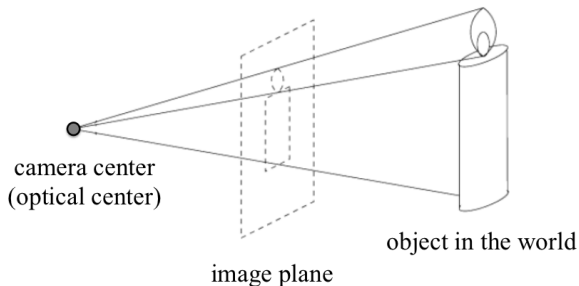
[Pics from: A. Torralba, Forsyth & Ponce]

- We will use the pinhole model as an approximation



- Since it's easier to think in a non-upside-down world, we will work with the virtual image plane, and just call it the image plane.
- How do points in 3D project to image plane? If I know a point in 3D, can I compute to which pixel it projects?

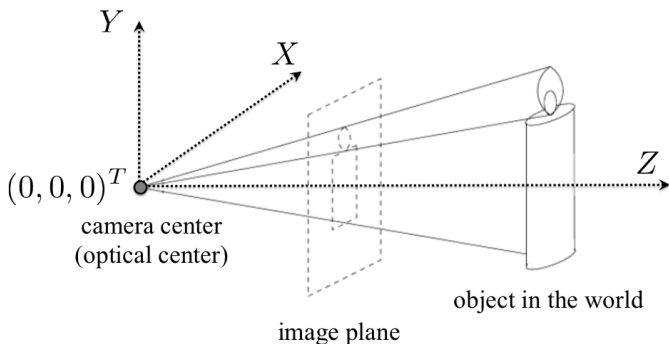
Modeling Projection



- First some notation which will help us derive the math
- To start with, we need a coordinate system

Modeling Projection

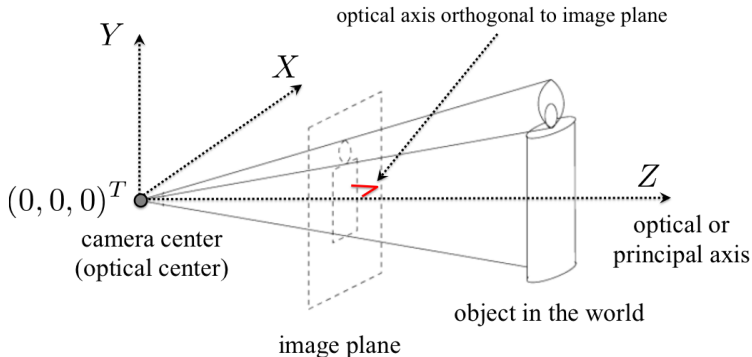
camera coordinate system in 3D



- We place a coordinate system relative to camera: **optical center** or **camera center \mathbf{C}** is thus at origin $(0, 0, 0)$.

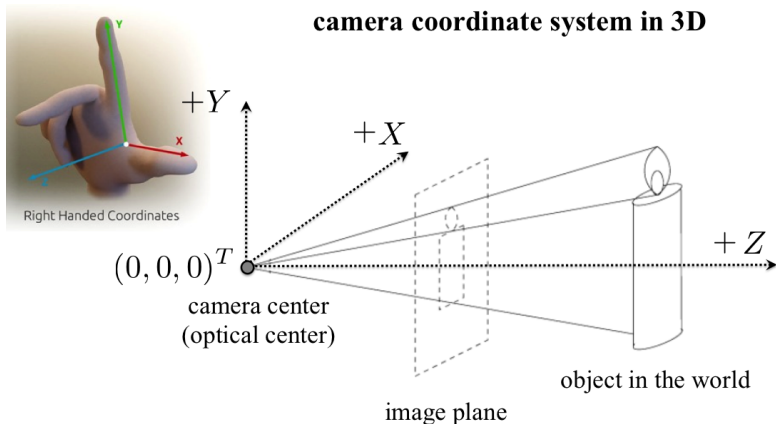
Modeling Projection

camera coordinate system in 3D



- The **Z** axis is called the **optical** or **principal axis**. It is orthogonal to the image plane. Axes **X** and **Y** are parallel to the image axes.

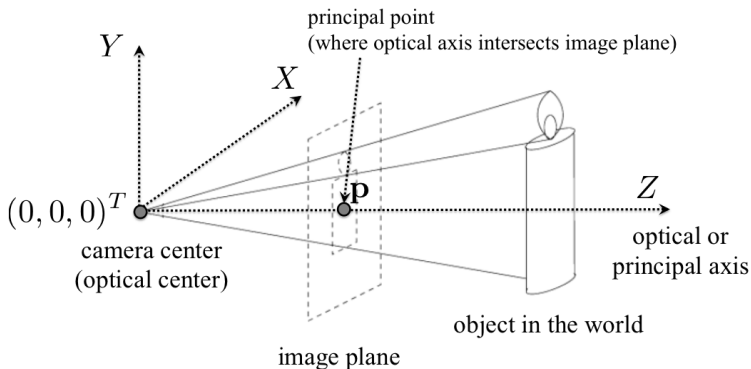
Modeling Projection



- We will use a **right handed** coordinate system

Modeling Projection

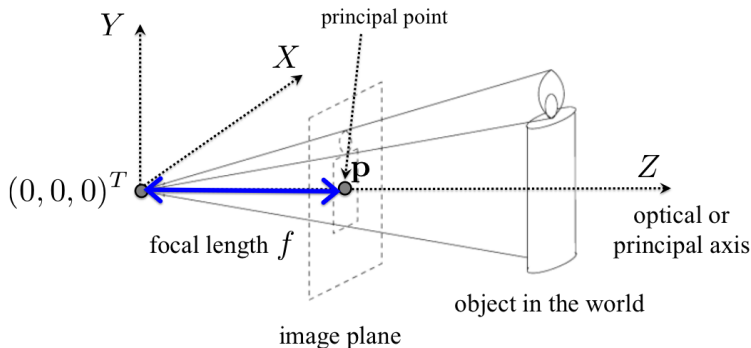
camera coordinate system in 3D



- The optical axis intersects the image plane in a point, \mathbf{p} . We call this point a **principal point**. It's worth to remember the principal point since it will appear again later in the math.

Modeling Projection

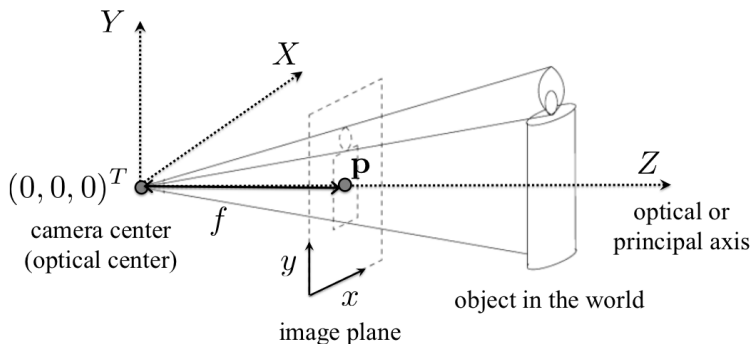
camera coordinate system in 3D



- The distance from the camera center to the principal point is called **focal length**, we will denote it with f . It's worth to remember the focal length since it will appear again later in the math.

Modeling Projection

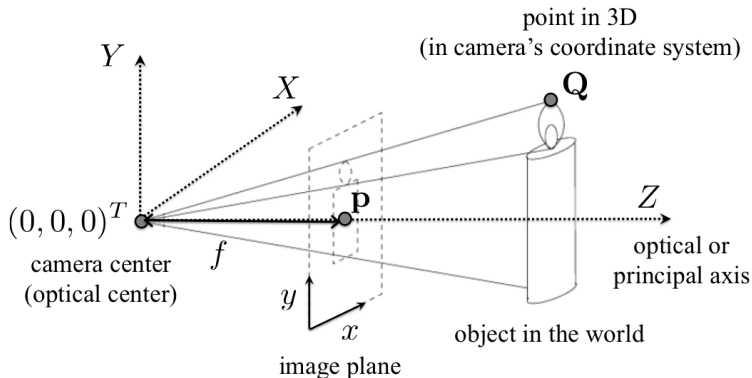
camera coordinate system in 3D



- We'll denote the image axes with x and y . An image we see is of course represented with these axes. We'll call this an **image coordinate system**.
- The tricky part is how to get from the camera's coordinate system (3D) to the image coordinate system (2D).

Modeling Projection

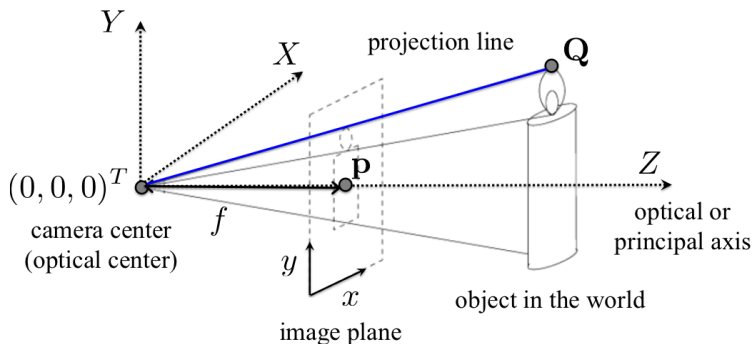
camera coordinate system in 3D



- Let's take some point Q in 3D. Q "lives" relative to the camera; its coordinates are assumed to be in camera's coordinate system.

Modeling Projection

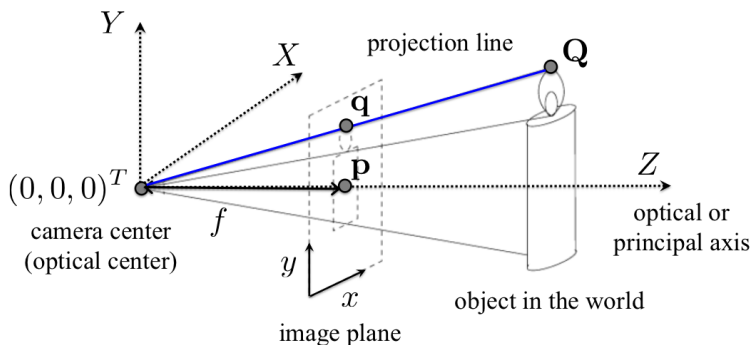
camera coordinate system in 3D



- We call the line from Q to camera center a **projection line**.

Modeling Projection

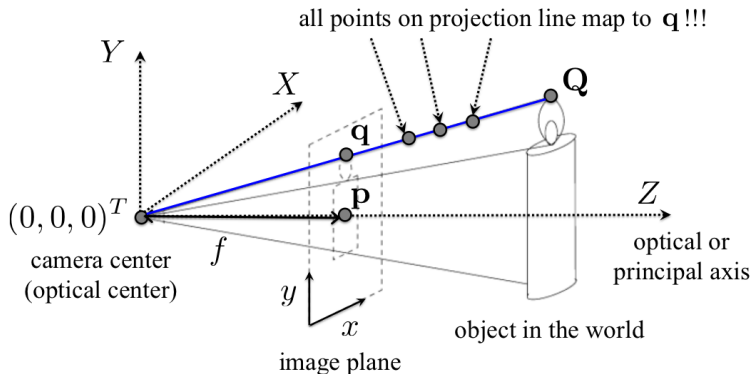
camera coordinate system in 3D



- The projection line intersects the image plane in a point q . This is the point we see in our image.

Modeling Projection

camera coordinate system in 3D



- First thing to notice is that all points from Q 's projection line project to the same point q in the image!
- **Ambiguity:** It's impossible to know how far a 3D point is from the camera along the projection line by looking only at the image (point q).

Modeling projection



From the movie Bone Collector

- **Ambiguity:** It's impossible to know how far a 3D point is from the camera along the projection line by looking only at the image (point \mathbf{q}).
- It's impossible to know the real 3D size of objects just from an image
- Why did the detective put a dollar bill next to the footprint?
- How would you compute the shoe's dimensions?

Modeling projection

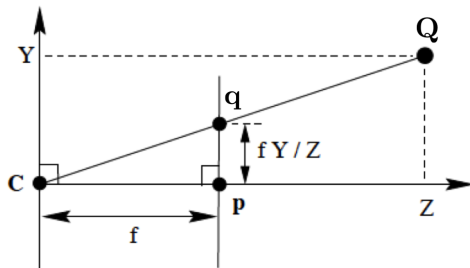


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Projection: Ready for Math

[Pic from: Zisserman & Hartley]



Projection Equations

- Using similar triangles:

$$\mathbf{Q} = (X, Y, Z)^T \rightarrow \left(\frac{f \cdot X}{Z}, \frac{f \cdot Y}{Z}, f \right)^T$$

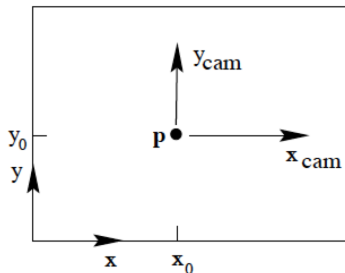
- This is relative to principal point \mathbf{p} . To move the origin to $(0,0)$ in image:

$$\mathbf{q} = (X, Y, Z)^T \rightarrow \left(\frac{f \cdot X}{Z} + p_x, \frac{f \cdot Y}{Z} + p_y, f \right)^T$$

where $\mathbf{p} = (p_x, p_y)$ is the principal point

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- Get the projection by throwing the last coordinate:

$$\mathbf{Q} = (X, Y, Z)^T \rightarrow \mathbf{q} = \left(\frac{f \cdot X}{Z} + p_x, \frac{f \cdot Y}{Z} + p_y \right)^T$$

- This is NOT a linear transformation as a division by Z is non-linear

Homogeneous Coordinates!

- We will use homogeneous coordinates, which simply append a 1 to the vector

Homogeneous coordinates to the rescue!

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

homogeneous image
coordinates

$$(x, y, z) \Rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

homogeneous scene
coordinates

Converting *from* homogeneous coordinates

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w)$$

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \Rightarrow (x/w, y/w, z/w)$$

[Source: N. Snavely]

Homogeneous Coordinates!

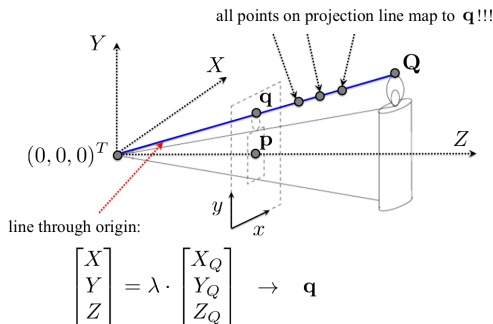
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- In Projective Geometry, all points are equal under scaling

Useful Trivia about Homogeneous Coordinates

- Homogeneous coordinates are quite useful in general. Let's see why
- Let's look at equation of a line in 2D: $ax + by + c = 0$
- I can represent the line with a homogeneous vector $\mathbf{l} := (a, b, c)^T$ (why homogeneous?) and a homogeneous vector $\mathbf{x} := (x, y, 1)$. How?

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- So if I have a line and someone gives me a point (x, y) , how do I quickly check if the point lies on the line? Homogeneous coordinates, and check if dot product is 0.
- Ok, what if I give you two points (x_1, y_1) and (x_2, y_2) and I ask you to write me an equation for the line between them?

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Back to Perspective Projection

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$$\mathbf{Q} = (X, Y, Z)^T \rightarrow \mathbf{q} = \begin{bmatrix} \frac{f \cdot X}{Z} + p_x \\ \frac{f \cdot Y}{Z} + p_y \end{bmatrix}$$

- Write this with homogeneous coordinates:

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Camera Intrinsics

- From previous slide:

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- Write:

$$K = \begin{bmatrix} f & 0 & p_x \\ 0 & f & p_y \\ 0 & 0 & 1 \end{bmatrix}$$

This is called a **camera calibration matrix** or **intrinsic parameter matrix**.
The parameters in K are called **internal camera parameters**.

- Finally: $\begin{bmatrix} w \cdot x \\ w \cdot y \\ w \end{bmatrix} = K \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \rightarrow \mathbf{q} = \begin{bmatrix} x \\ y \end{bmatrix}$

[Source: Zisserman & Hartley]

Camera Intrinsics

- From previous slide:

$$\mathbf{Q} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \rightarrow \begin{bmatrix} f \cdot X + Z \cdot p_x \\ f \cdot Y + Z \cdot p_y \\ Z \end{bmatrix} = \begin{bmatrix} f & 0 & p_x \\ 0 & f & p_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

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[Source: Zisserman & Hartley]

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- And there might be a skew angle θ between x and y image axis:

$$K = \begin{bmatrix} f_x & -f_x \cot \theta & p_x \\ 0 & f_y / \sin \theta & p_y \\ 0 & 0 & 1 \end{bmatrix}$$

[Source: Zisserman & Hartley]

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We'll work with this one

- It can be a little more complicated. Pixels may not be square:

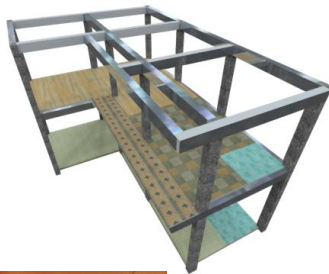
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[Source: Zisserman & Hartley]

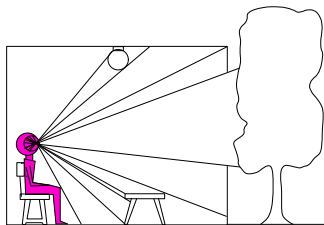
Perspective Projection



[Source: N. Snavely]

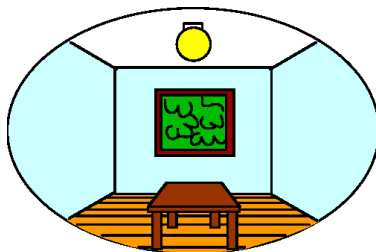
Dimensionality Reduction Machine (3D to 2D)

3D world



Point of observation

2D image



What have we lost?

- Angles
- Distances (lengths)

Slide by A. Efros

Figures © Stephen E. Palmer, 2002

Projection properties

- **Many-to-one:** any points along same ray map to same point in image

Projection properties

- **Many-to-one:** any points along same ray map to same point in image
- Points \rightarrow points

Projection properties

- **Many-to-one:** any points along same ray map to same point in image
- Points \rightarrow points
- Lines \rightarrow lines. Why?



Projection properties

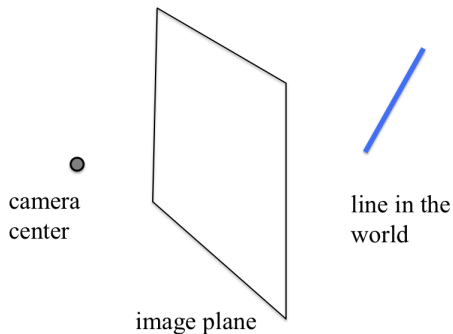


Figure: Proof by drawing

Projection properties

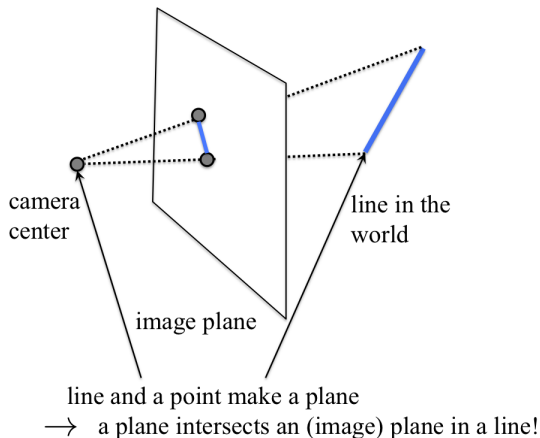


Figure: Proof by drawing

Projection properties

- **Many-to-one:** any points along same ray map to same point in image
- Points \rightarrow points
- Lines \rightarrow lines
- But line through principal point projects to a point. Why?



Figure: Can you tell where is the principal point?

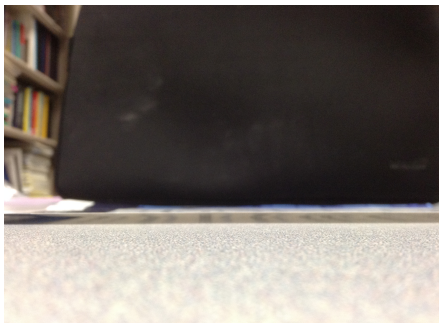
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Projection properties

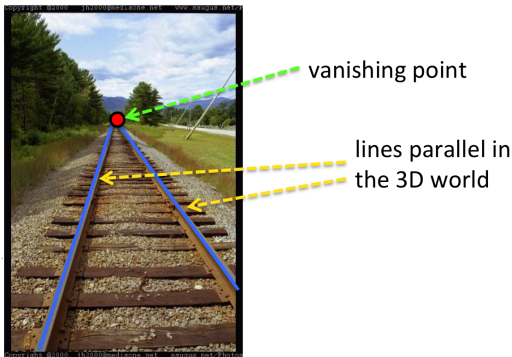
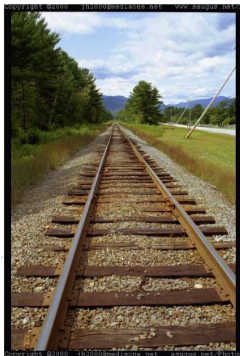
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- But line through principal point projects to a point. Why?
- Planes \rightarrow planes
- But plane through principal point projects to line. Why?



Projection Properties: Cool Facts

Parallel lines converge at a **vanishing point**

- Each different direction in the world **has its own vanishing point**

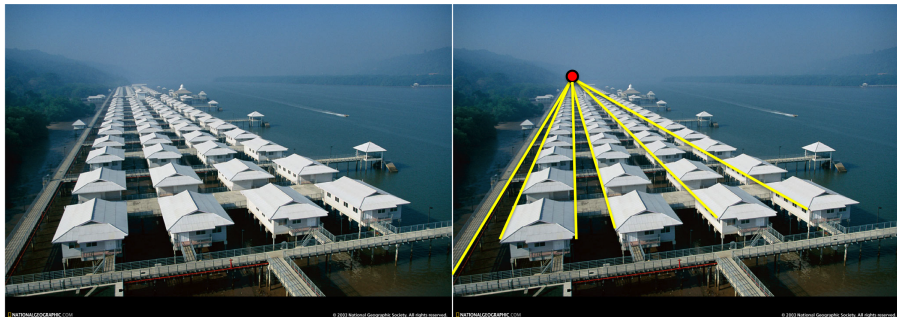


[Adopted from: N. Snavely, R. Urtasun]

Projection Properties: Cool Facts

Parallel lines converge at a **vanishing point**

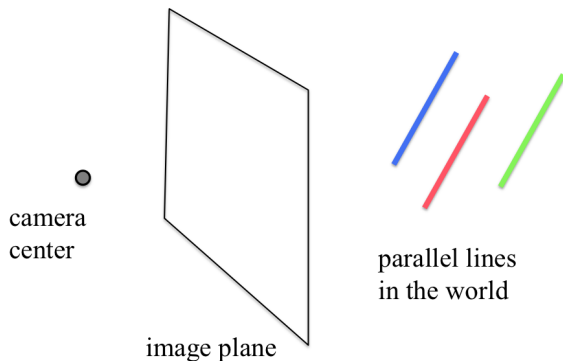
- Each different direction in the world **has its own vanishing point**
- All lines with the same 3D direction intersect at the **same vanishing point**



[Pic: R. Szeliski]

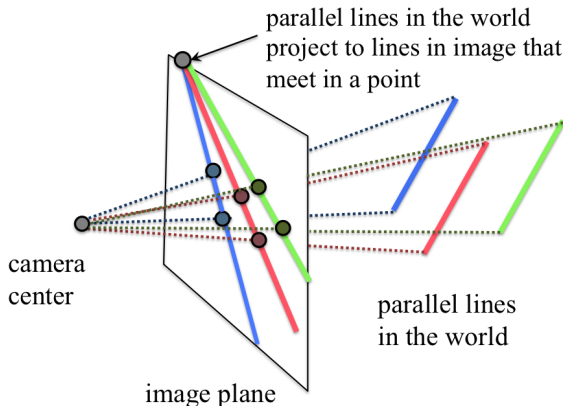
Projection Properties: Vanishing Point

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Why?



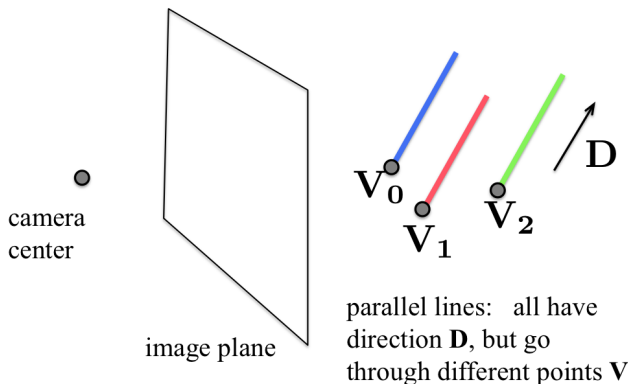
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Projection Properties: Vanishing Point

- All lines with the same 3D direction intersect at the **same vanishing point**.
Why?
- Line that passes through \mathbf{V} with direction \mathbf{D} : $\mathbf{X} = \mathbf{V} + t\mathbf{D}$.
- Project it:

$$\begin{bmatrix} wx \\ wy \\ w \end{bmatrix} = K\mathbf{X} = \begin{bmatrix} f & 0 & p_x \\ 0 & f & p_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_x + tD_x \\ V_y + tD_y \\ V_z + tD_z \end{bmatrix} = \begin{bmatrix} fV_x + ftD_x + p_x V_z + tp_x D_z \\ fV_y + ftD_y + p_y V_z + tp_y D_z \\ V_z + tD_z \end{bmatrix}$$

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- Send $t \rightarrow \infty$ and compute x and y :

$$x = \lim_{t \rightarrow \infty} \frac{fV_x + ftD_x + p_x V_z + tp_x D_z}{V_z + tD_z} = \frac{fD_x + p_x D_z}{D_z}$$
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- This doesn't depend on \mathbf{V} ! So all lines with direction \mathbf{D} go to this point!

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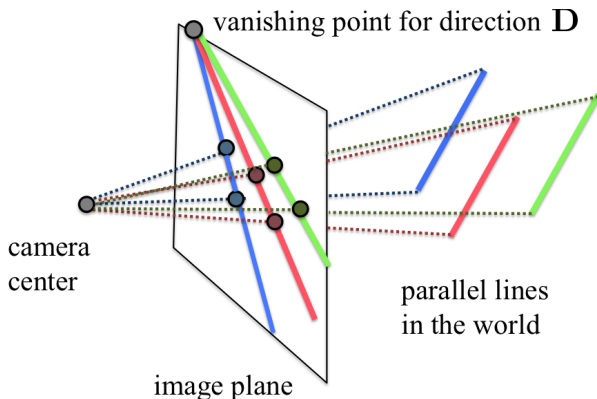
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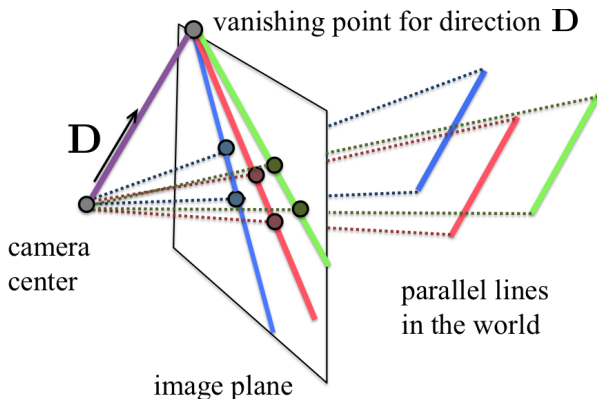
Projection Properties: Vanishing Point

- All lines with the same 3D direction intersect at the **same vanishing point**.
- The easiest way to find this point: Translate line with direction \mathbf{D} to the camera center. This line intersects the image plane in the vanishing point corresponding to direction \mathbf{D} ! Why?



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Projection Properties: Cool Facts

Parallel lines converge at a **vanishing point**

- Each different direction in the world **has its own vanishing point**
- Lines parallel to image plane are also parallel in the image (we say that they intersect at infinity). Why?

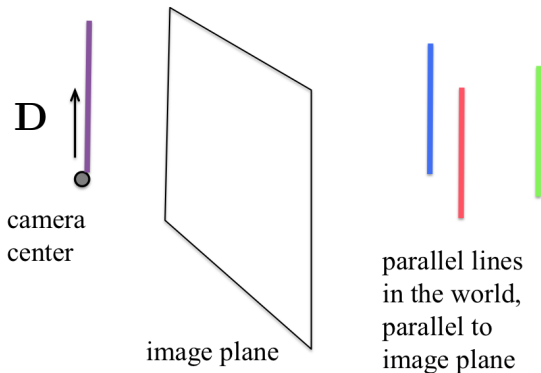


In 3D: parallel to image plane
In 2D: parallel (no VP)

Projection Properties: Cool Facts

- Lines parallel to image plane are also parallel in the image. We say that they intersect at infinity.

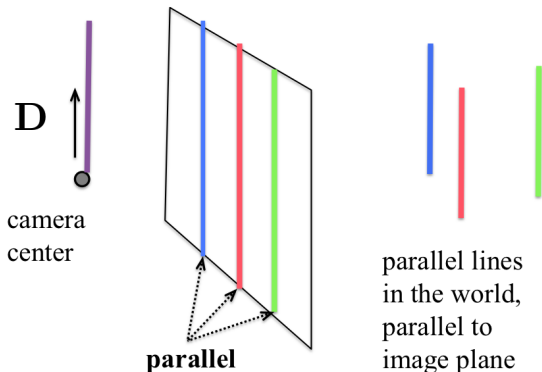
doesn't intersect image plane! So no vanishing point!



Projection Properties: Cool Facts

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doesn't intersect image plane! So no vanishing point!



Projection Properties: Cool Tricks

- This picture has been recorded from a car with a camera on top. We know the camera intrinsic matrix K .
- Can we tell the incline of the hill we are driving on?
- How?



Projection Properties: Cool Tricks

- Can we tell the incline of the hill we are driving on?

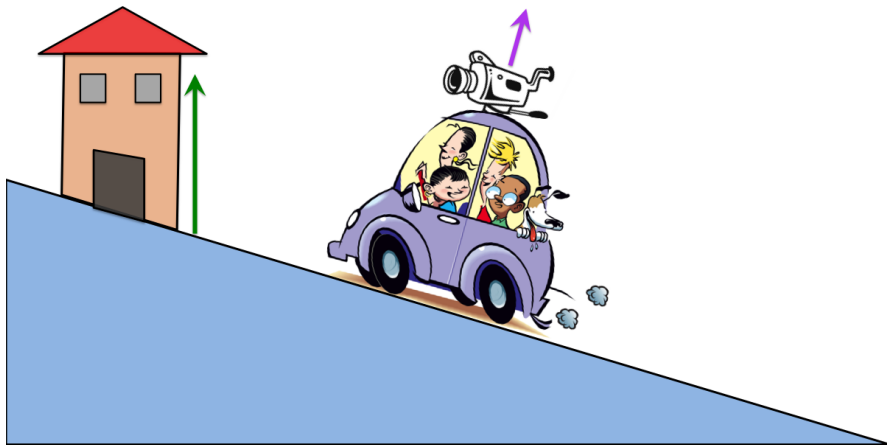


Figure: This is the 3D world behind the picture.

Projection Properties: Cool Tricks

- Can we tell the incline of the hill we are driving on?

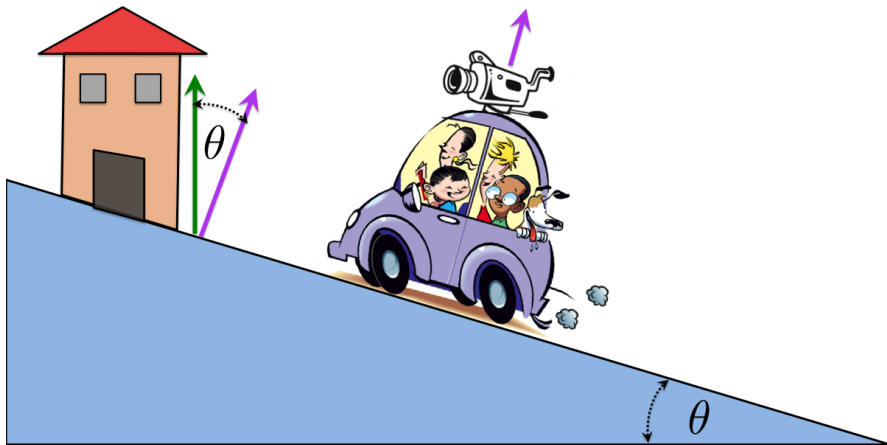


Figure: If we compute the 3D direction of the house's vertical lines relative to camera, we have the incline! How can we do that?

Projection Properties: Cool Tricks

- Can we tell the incline of the hill we are driving on?

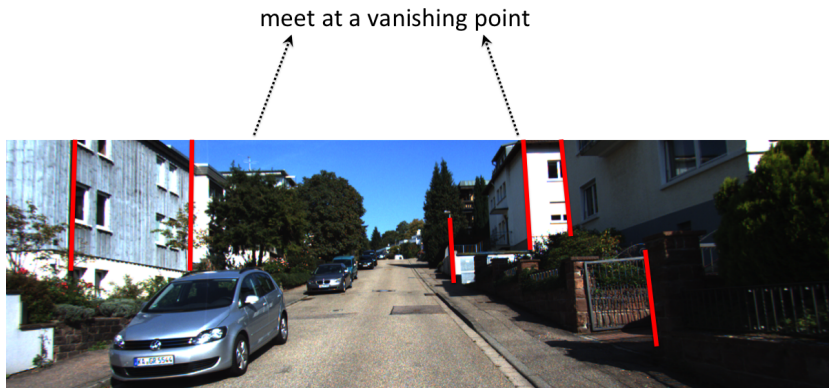


Figure: Extract “vertical” lines and compute vanishing point. How can we compute direction in 3D from vanishing point (if we have K)?

Projection Properties: Cool Tricks

- Can we tell the incline of the hill we are driving on?

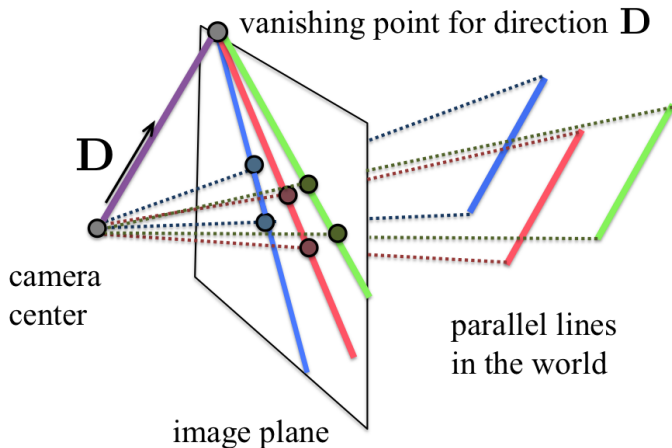
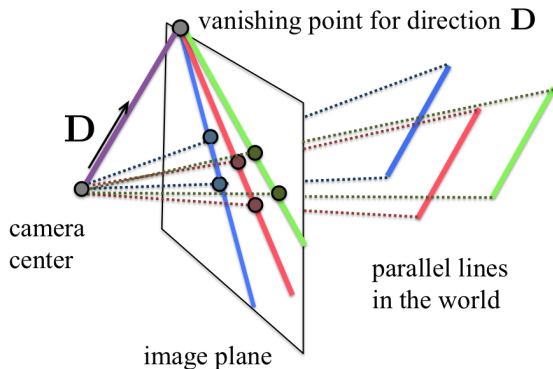


Figure: This picture should help.

Projection Properties: Cool Tricks

- Can we tell the incline of the hill we are driving on?

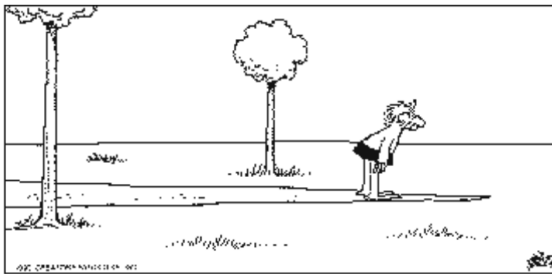
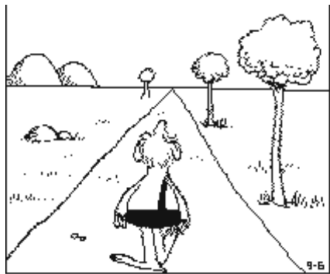


- We have:

$$\begin{bmatrix} w \cdot vp_x \\ w \cdot vp_y \\ w \end{bmatrix} = K\mathbf{D} \quad \rightarrow \quad \mathbf{D} = wK^{-1} \begin{bmatrix} vp_x \\ vp_y \\ 1 \end{bmatrix} \quad \rightarrow \quad \text{normalize } \mathbf{D} \text{ to norm 1}$$

Vanishing Points Can be Deceiving

- Parallel lines converge at a **vanishing point**.
- But intersecting lines in 2D are not necessary parallel in 3D.

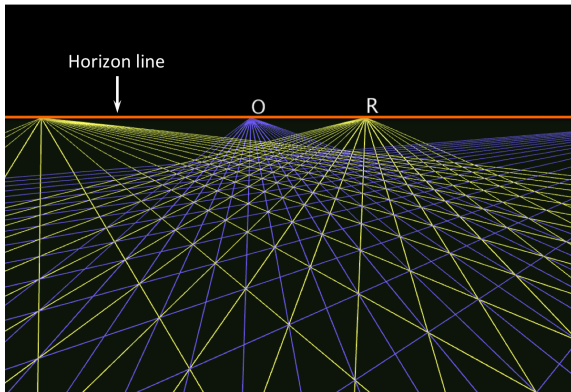


[Source: A. Jepson]

Projection Properties: Cool Facts

Parallel lines converge at a **vanishing point**

- Each different direction in the world **has its own vanishing point**
- For lines on the same 3D plane, the vanishing points lie on a **line**. We call it a **vanishing line**. Vanishing line for the ground plane is a **horizon line**.



<http://4.bp.blogspot.com/-0Jm9d9j35Tc/T5ESbVpKI7I/AAAAAAAAACEk/nVAiTxBuiyc/s1600/perspectiveGrid-01.png>

Projection Properties: Cool Facts

Parallel lines converge at a **vanishing point**

- For lines on the same 3D plane, the vanishing points lie on a **line**. We call it a **vanishing line**. Vanishing line for the ground plane is a **horizon line**.
- Some horizon lines are nicer than others ;)



Punta Cana

Projection Properties: Cool Facts

Parallel lines converge at a **vanishing point**

- For lines on the same 3D plane, the vanishing points lie on a **line**. We call it a **vanishing line** or a **horizon line**.
- Parallel planes in 3D have the **same horizon line** in the image.



Projection Properties: Cool Facts

- Can I tell how much above ground this picture was taken?



Projection Properties: Cool Facts

- Can I tell how much above ground this picture was taken?



Projection Properties: Cool Facts

- Same distance as where the horizon intersects a building



Projection Properties: Cool Facts

- Same distance as where the horizon intersects a building: 50 floors up



Projection Properties: Cool Facts

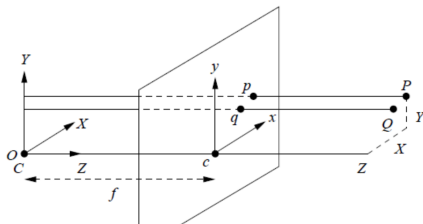
- This is only true when the camera (image plane) is orthogonal to the ground plane. And the ground plane is flat.
- A very nice explanation of this phenomena can be found by Derek Hoiem here:

https://courses.engr.illinois.edu/cs543/sp2011/materials/3dscene_book_svg.pdf



Orthographic Projection

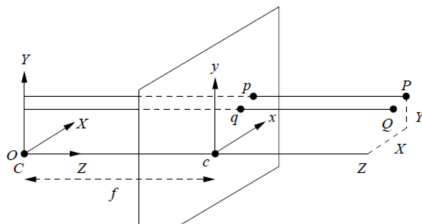
[Source: R. Urtasun]



- Requires no division and simply drops the Z coordinate.

Orthographic Projection

[Source: R. Urtasun]

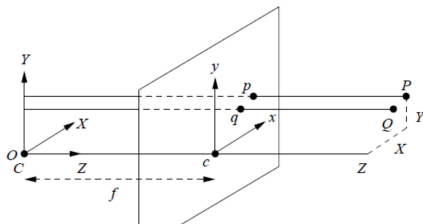


- Requires no division and simply drops the Z coordinate.
- Orthographic projection:

$$\mathbf{Q} = \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} \rightarrow \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Orthographic Projection

[Source: R. Urtasun]



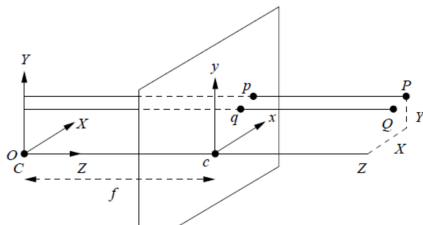
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- Special case of perspective projection where the distance from the camera center to the image plane is infinity

Orthographic Projection

[Source: R. Urtasun]

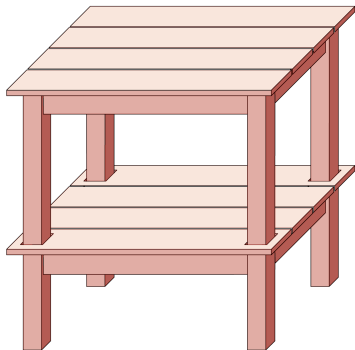


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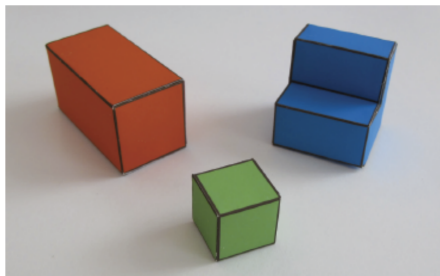
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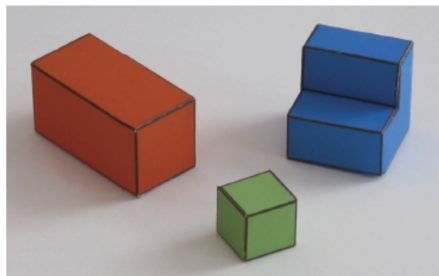
Orthographic Projection



Orthographic Projection



Perspective projection



Parallel (orthographic) projection

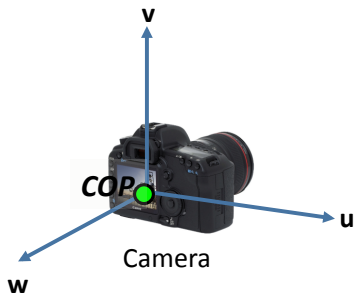
- For perspective projection lines parallel in 3D **are not** parallel in the image.
- For orthographic projection lines parallel in 3D **are** parallel in the image.

[Source: A. Torralba]

Camera Parameters

We are not yet done with projection. To fully specify projection, we need to:

- Describe its **internal parameters** (we know this, this is our \mathbf{K})
- Describe its **pose in the world**. Two important coordinate systems:
 - World coordinate system
 - Camera coordinate system

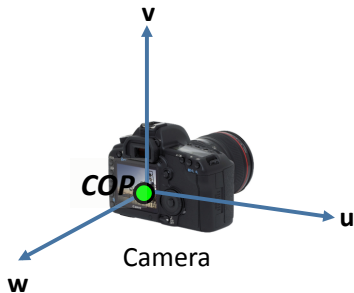


[Source: N. Snavely, slide credit: R. Urtasun]

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[Source: N. Snavely, slide credit: R. Urtasun]

Camera Parameters

- Why two coordinate systems?



Figure: Imagine this is your room.

Camera Parameters

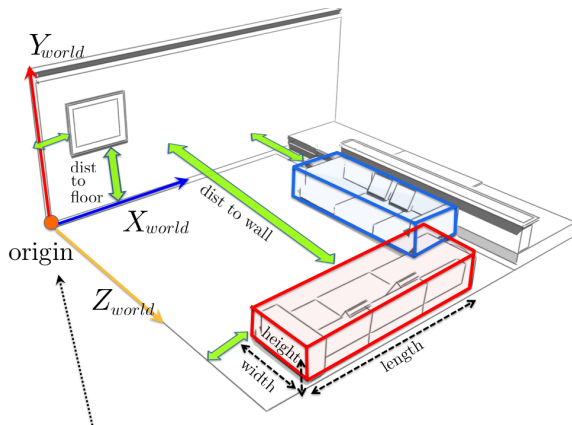
- Why two coordinate systems?



Figure: When you were furnishing you measured everything in detail.

Camera Parameters

- Why two coordinate systems?

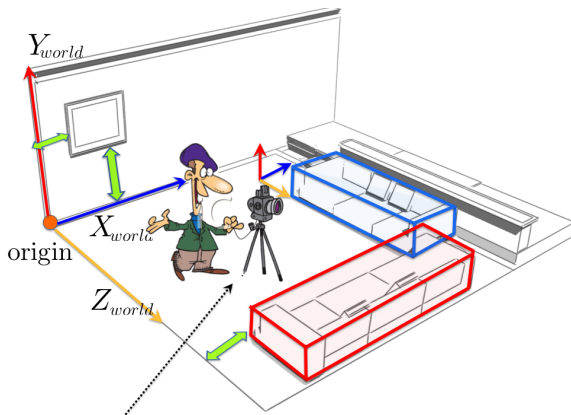


I measured everything in my room relative to this point

Figure: Thus you know all coordinates relative to a special point (origin) and coordinate system in the room. This is your **room's (world) coordinate system**.

Camera Parameters

- Why two coordinate systems?



But to project my room to camera, I need to have the room in the camera coordinate system!

Figure: Now you take a picture and you wonder how points project to camera. In order to project, you need all points in **camera's coordinate system**.

Projection

To project a point (X, Y, Z) in world coordinates on the image plane, we need to:

- Transform (X, Y, Z) into camera coordinates. We thus need:

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[Source: N. Snavely, slide credit: R. Urtasun]

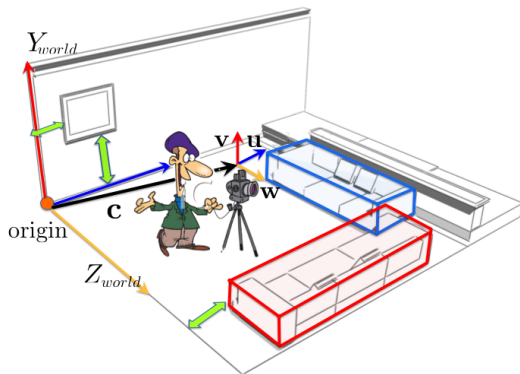
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Camera Extrinsics

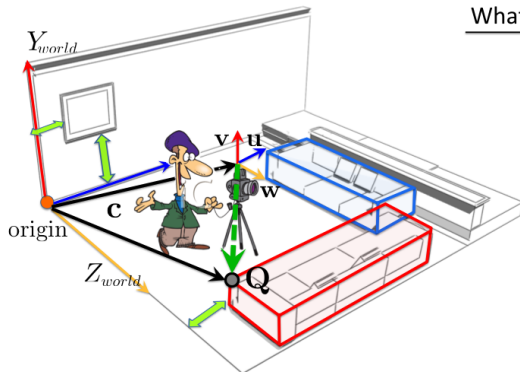


- c ... camera position in room coordinate system
- u, v, w ... 3 orthogonal directions of camera in room coordinate system

Figure: We first need our camera position and orientation in the room's world.

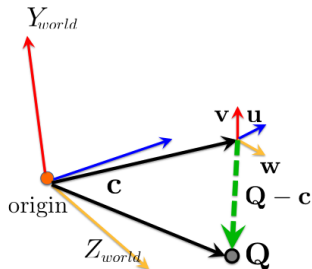
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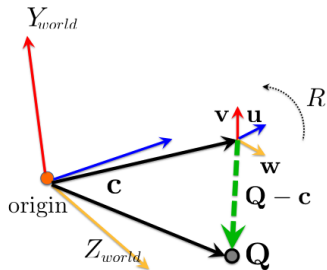


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$Q - c$... makes **position** relative to camera

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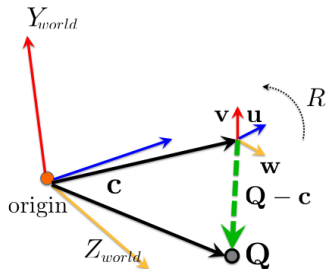
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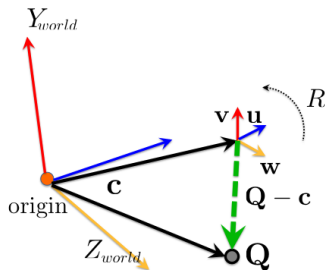
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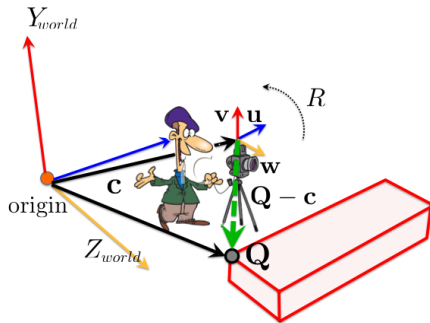
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$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = R \left(\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} - \mathbf{c} \right) = \begin{bmatrix} R & -R\mathbf{c} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

camera coordinates

room (world) coordinates

Figure: Final Transformation

Projection Equations

- **Projection matrix** P models the cumulative effect of all intrinsic and extrinsic parameters. We use homogeneous coordinates for 2D and 3D:

$$\mathbf{q} = \begin{bmatrix} ax \\ ay \\ a \end{bmatrix} = \mathbf{P} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

- It can be computed as

$$\mathbf{P} = \underbrace{\begin{bmatrix} f & 0 & p_x \\ 0 & f & p_y \\ 0 & 0 & 1 \end{bmatrix}}_{\text{intrinsic } K} \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{\text{projection}} \underbrace{\begin{bmatrix} \mathbf{R}_{3 \times 3} & \mathbf{0}_{3 \times 1} \\ \mathbf{0}_{1 \times 3} & 1 \end{bmatrix}}_{\text{rotation}} \underbrace{\begin{bmatrix} \mathbf{I}_{3 \times 3} & \mathbf{T}_{3 \times 1} \\ \mathbf{0}_{1 \times 3} & 1 \end{bmatrix}}_{\text{translation}}$$

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It's the same thing.

- This might look complicated. But the truth is, in most cases you don't have \mathbf{P} at all, so you can't really compute any projections. When you have a calibrated camera, then someone gives you \mathbf{P} . And then projection is easy.

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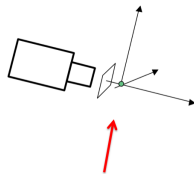
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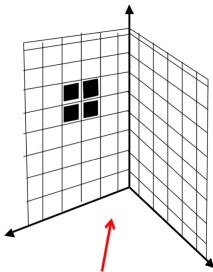
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A Short Note on Camera Calibration



Detect corners in image and figure out which corner corresponds to which point in the 3D pattern.



You measured all distances for this pattern.

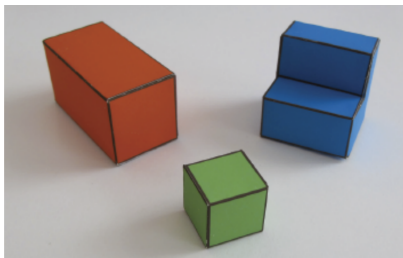
The general procedure:

- Place a 3D pattern (for which you know all distances) in front of camera.
- Take a picture. Detect corners in image and find correspondences to the points in the pattern.
- Go to the internet and check out the math that tells you how to compute K from these 2D-3D correspondences. ;) We won't cover in class.

[Pic from: R. Duraiswami]

Camera Calibration: Interesting Fact

- Let's say you have an image but you don't know **anything** about the camera (for example, image downloaded from the web).
- For images where you see lines corresponding to 3 orthogonal directions, like cubes or rooms, you can compute the camera matrix K as well as R and t !



- How to do this is explained in the Zisserman & Hartley book.

Projection Properties: Cool Facts

- As a consequence, for scenes with lots of lines (e.g. man-made scenes) one can reconstruct the scene in 3D from a **single image**!



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Projection Properties: Cool Facts

- As a consequence, for scenes with lots of lines (e.g. man-made scenes) one can reconstruct the scene in 3D from a **single image**!
- For those interested, check out the math here:

A. Criminisi, I. Reid, and A. Zisserman

Single View Metrology

International Journal of Computer Vision, vol 40, num 2, 2000

<http://www.cs.cmu.edu/~ph/869/papers/Criminisi99.pdf>

Camera Calibration: Another Interesting Fact



- From a longer **video** in which the sun travels across the sky you can compute the camera intrinsic matrix, as well as extrinsic, i.e., the GPS location where you are! Well, up to a 100km accuracy...

Camera Calibration: Another Interesting Fact

J.-F. Lalonde, S. G. Narasimhan, and A. A. Efros

What Do the Sun and Sky Tell Us About the Camera?

International Journal on Computer Vision, 88(1), May 2010

Paper: <http://vision.gel.ulaval.ca/~jflalonde/projects/sky/index.html>

Code: <https://github.com/jflalonde/webcamCalibration>

- From a longer **video** in which the sun travels across the sky you can compute the camera intrinsic matrix, as well as extrinsic, i.e., the GPS location where you are! Well, up to a 100km accuracy...
- Is this useful? Maybe, to catch terrorists that record their videos outside.

Exercise (Not Very Easy, But Fun)

- We want to render (project) a 3D CAD model of a car to this image in a realistic way
- How?



Exercise

- First get a CAD model. There are tones of them, e.g. 3D Warehouse (free)

<https://3dwarehouse.com/search.html?q=car&backendClass=entity>

3D Warehouse

car

Search

Sign In

54,155 Results

ALL

Results Per Page

Sort by Relevance

Aerodynamic Car: Coupe 3.3i C...
by: Dm26

Concept Car
by: DUBERT

CAR-TOONY 5 / Fun Little Car
by: COL 1

B-type race spec car concept
by: Peter B.

Super-Duper Drivby Thing
by: MSL16E

Fururistik Yellow car
by: Small Po

LL Laurent GT 1972
by: l | Le-04-11

Custom Car - COL 1 Edition
by: COL 1

CAR-TOONY 3 / Fun Little Car
by: COL 1

google race car 3MB all done =...
by: *BABAAS*

CAR-TOONY 6 / Fun Little Car
by: COL 1

Sports Car
by: waty188

Crystal Shine Car Wash - Fully F...
by: Cordele

barosso 2.2 concept car
by: Peter B.

barosso b-type concept car
by: Peter B.

Yellow Car
by: (Dodge33) - AJ

Trimble

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Privacy

Terms of Use

English

Exercise

- We downloaded this model. Now what?

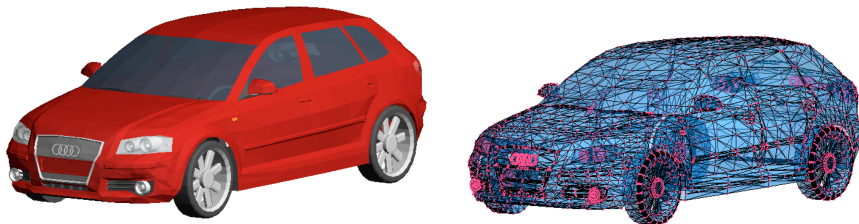


Figure: A CAD model is a collection of 3D vertices and faces that connect the vertices. Each face represents a small triangle. It typically has color.

Exercise

- Our image was collected with a car on the road:
 - A camera was on top of the car, approximately $1.7m$ above ground
 - Image plane is orthogonal to the ground
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- With a little bit of math, we can compute the ground plane in 3D, relative to camera. We a bit more math we can compute which point on the ground plane projects to an image point (x, y) .

Extra Credit: 3 points: How? ;) I will post online the code that does everything but this part.

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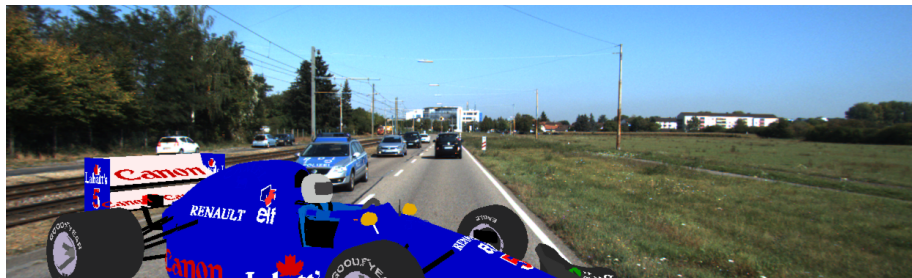
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- That's it. Make a video for more fun



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A Little More on Camera Models