Due: By 6pm on Thursday 6 December. **Worth**: 5%

Keep in mind that your proofs will be graded on their structure at least as much as on their content. In other words, when we ask you to prove a statement, it is to find out how well you can write proofs, not because we are interested in knowing whether or not that statement is true. So pay particular attention to writing your proofs properly.

1. [5 marks]

   Let R be any regular expression over alphabet Σ. Show that there exists a regular expression T over Σ such that $L(T) = \overline{L(R)}$ (the complement of the language of R).

   Note: you are not expected to describe T explicitly, just give a rigorous argument that it exists.

2. [15 marks]

   Consider the language $L = \{s \in \{0\}^* : \text{the length of } s \text{ is divisible by } 3\}$; for example, $\varepsilon \in L$, $000 \in L$, $000000000 \in L$, but $0 \notin L$, $00 \notin L$, $0000 \notin L$.

   (a) Give a 3-state FSA for L, *i.e.*, a FSA A such that $L(A) = L$ and A contains no more than 3 states. Justify briefly that your FSA is correct.

   (b) Prove that no FSA with 2 states can accept L, *i.e.*, for all FSA A with 2 states, $L(A) \neq L$.

   (c) Generalize to show that for every $k \in \mathbb{N}$, there is a language L such that L is accepted by some FSA with k states but not by any FSA with fewer than k states.

3. [10 marks]

   Let $L = \{s \in \{0,1\}^* : s \text{ is the binary repreesntation of a natural number divisible by } 3\}$; for example, $\varepsilon \in L$, $0 \in L$, $0011 \in L$, $1001 \in L$, but $1 \notin L$, $101 \notin L$, $1000 \notin L$. Show that L is regular—include a brief explanation of the correctness of your FSA or regular expression.

   HINT: What is the arithmetic relationship between the number represented by some binary string s and the numbers represented by the binary strings $s0$ and $s1$?