CSC165 Quiz 8, Thursday July 21st

Name: Student number:

Prove or disprove, using our formatted proof structure, and the definition of big-Oh from class,[1] the following claim:

$$(7n^3 + 11n^2 + n) \in O(n^3)$$

Sample solution: The claim is true.
Let $c = 8$. Let $B = 12$.

Then c is a positive real number and B is a natural number.

Let $n \in \mathbb{N}$. Assume $n \geq B$.

Then $n^3 = n \times n^2 \geq 12 \times n^2 = 11 \times n^2 + n^2$. (since $n \geq B = 12$).
So $n^2 \geq 12n$. (since $n \geq 12$, multiplying both sides by $n > 0$).
So $12 > 1 \Rightarrow 12n > n$. (Multiplying both sides by $n > 0$).
So $n^3 \geq 12n^2 = 11n^2 + n^2 \geq 11n^2 + 12n \geq 11n^2 + n$.
So $7n^3 \geq 7n^3$.
Thus $cn^3 = 8n^3 = 7n^3 + n^3 \geq 7n^3 + 11n^2 + n$. (adding the two inequalities).

So $n \geq B \Rightarrow 7n^3 + 11n^2 + n \leq cn^3$.

Since n is an arbitrary element of $\mathbb{N}$, $\forall n \in \mathbb{N}, n \geq B \Rightarrow 7n^3 + 11n^2 + n \leq cn^3$.

Since c is a real positive number and B is a natural number, $\exists c \in \mathbb{R}^+, \exists B \in \mathbb{N}, \forall n \in \mathbb{N}, n \geq B \Rightarrow 7n^3 + 11n^2 + n \leq cn^3$.
By definition, $(7n^3 + 11n^2 + n) \in O(n^3)$.

[1] $O(f) = \{g : \mathbb{N} \mapsto \mathbb{R}^{\geq 0} | \exists c \in \mathbb{R}^+, \exists B \in \mathbb{N}, \forall n \in \mathbb{N}, n \geq B \Rightarrow g(n) \leq cf(n)\}$