

- 0 There are four cards on a table showing symbols D, E, 2, and 3 (one per card). Each card has a letter on one side and a digit on the other. Which card(s) do you need to turn over to determine whether every card with a D on one side has a 3 on the other? Why?

After trying the question, scroll down to the solution.

Solutions

- § You may have to turn over both D and 2, or you may have your answer after turning over just one of them. If the D has anything other than 3 on the other side, you answer no. If the 2 has a D on the other side, you answer no. If the D has a 3 on the other side, and the 2 doesn't have a D on the other side, then you answer yes. It doesn't matter what the E has on the other side. As for the 3, if it has a D and you already answered yes, this won't change your answer. If it has a D and you already answered no because you previously saw a D without a 3, again this won't change your answer. And if the 3 has anything other than a D on the other side, then again it doesn't change your answer about what Ds have on their other sides.

The question does not ask for formalization, and that's good, because in Chapter 0 we haven't yet seen how to formalize. But for later reference, we can formalize the problem as follows. Let's refer to the top (visible) sides of the cards as t_0 , t_1 , t_2 , and t_3 , and the bottom (invisible) sides of the same cards as b_0 , b_1 , b_2 , and b_3 respectively. Part of the given information is

$$t_0 = \text{“D”} \wedge t_1 = \text{“E”} \wedge t_2 = \text{“2”} \wedge t_3 = \text{“3”}$$

Another part of the given information is

$$b_0 \neq \text{“D”} \wedge b_1 \neq \text{“D”} \wedge b_2 \neq \text{“3”} \wedge b_3 \neq \text{“3”}$$

because cards don't have letters on both sides, and they don't have digits on both sides.

The question we are asked is

$$\begin{aligned} & (t_0 = \text{“D”} \Rightarrow b_0 = \text{“3”}) \wedge (b_0 = \text{“D”} \Rightarrow t_0 = \text{“3”}) \\ \wedge \quad & (t_1 = \text{“D”} \Rightarrow b_1 = \text{“3”}) \wedge (b_1 = \text{“D”} \Rightarrow t_1 = \text{“3”}) \\ \wedge \quad & (t_2 = \text{“D”} \Rightarrow b_2 = \text{“3”}) \wedge (b_2 = \text{“D”} \Rightarrow t_2 = \text{“3”}) \\ \wedge \quad & (t_3 = \text{“D”} \Rightarrow b_3 = \text{“3”}) \wedge (b_3 = \text{“D”} \Rightarrow t_3 = \text{“3”}) \end{aligned}$$

Now use the given information

$$\begin{aligned} = & (\top \Rightarrow b_0 = \text{“3”}) \wedge (\perp \Rightarrow \perp) \\ \wedge \quad & (\perp \Rightarrow b_1 = \text{“3”}) \wedge (\perp \Rightarrow \perp) \\ \wedge \quad & (\perp \Rightarrow \perp) \wedge (b_2 = \text{“D”} \Rightarrow \perp) \\ \wedge \quad & (\perp \Rightarrow \perp) \wedge (b_3 = \text{“D”} \Rightarrow \top) \end{aligned}$$

Now simplify using the laws $(\top \Rightarrow a) = a$ and $(a \Rightarrow \perp) = \neg a$ and $(\perp \Rightarrow a) = \top$

$$= b_0 = \text{“3”} \wedge \top \wedge \top \wedge \top \wedge \top \wedge b_2 \neq \text{“D”} \wedge \top \wedge \top$$

Now simplify further using the laws $(\top \wedge a) = a$ and $(a \wedge \top) = a$

$$= b_0 = \text{“3”} \wedge b_2 \neq \text{“D”}$$

To answer the question with “yes”, we need to know that the bottom of card 0 is “3” and that the bottom of card 2 is not “D”. To answer the question with “no” we need to know either that the bottom of card 0 is not “3” or that the bottom of card 2 is “D”.

Here is a logically equivalent puzzle. Four people are in a bar. The first is drinking beer; the next one is drinking Coke; the next one is 16 years old; the last one is 25 years old. The law says that no-one under 21 years old is allowed to drink beer. To check whether the law is being followed, whom do we need to investigate further?