117 Let n be a natural number, and let R be a relation on $0,..n$. In other words,

R: $(0,..n) \to (0,..n) \to bin$

We say that from x we can reach x in zero steps. If $R\ x\ y$ we say that from x we can reach y in one step. If $R\ x\ y$ and $R\ y\ z$ we say that from x we can reach z in two steps. And so on. Express formally that from x we can reach y in some number of steps.

After trying the question, scroll down to the solution.

§ $x=y \ \vee\ \exists s{:}\ {*}nat\cdot\ \forall n{:}\ 0,..\#[s]+1\cdot\ R\ ([x;\ s;\ y]\ n)\ ([x;\ s;\ y]\ (n+1))$

Here is another solution. I omit domains, which are always $0,..n$. Define the relational composition $(R.\,S)$ of relations R and S as follows:

$$R.\,S \ = \ \langle x, y\cdot\ \exists z\cdot\ R\ x\ z \wedge S\ z\ y\rangle$$

Now define relational power R^m for relation R and natural m as follows:

$$R^0 = \langle x, y\cdot\ x=y\rangle \qquad \text{(the identity relation)}$$

$$R^{m+1} \ = \ R^m.\,R$$

Then $R^m\ x\ y$ says that from x we can reach y in m steps, and $\exists m\cdot\ R^m\ x\ y$ says that from x we can reach y in some number of steps.