136   Let x and y be binary variables. Simplify

(a) $x:=\ x=y.\ \ x:=\ x=y$

(b) $x:=\ x{+}y.\ \ y:=\ x{+}y.\ \ x:=\ x{+}y$

After trying the question, scroll down to the solution.

§(a) $x:= x=y. \ x:= x=y$ rewrite last assignment
   $=$ $x:= x=y. \ x' = (x=y) \wedge y'=y$ Substitution Law
   $=$ $x'=((x=y)=y) \wedge y'=y$ Associative Law for binary $=$
   $=$ $x'=(x=(y=y)) \wedge y'=y$ Reflexive and Identity laws for $=$
   $=$ $x'=x \wedge y'=y$
   $=$ ok

§(b) $x:= x{+}y. \ y:= x{+}y. \ x:= x{+}y$ rewrite last assignment
   $=$ $x:= x{+}y. \ y:= x{+}y. \ x' = (x{+}y) \wedge y'=y$ Substitution Law
   $=$ $x:= x{+}y. \ x'=(x{+}(x{+}y)) \wedge y'=(x{+}y)$ Substitution Law
   $=$ $x'=((x{+}y){+}((x{+}y){+}y)) \wedge y'=((x{+}y){+}y)$ Associativity and Symmetry of $+$
   $=$ $x'=(((x{+}x){+}(y{+}y)){+}y) \wedge y'=(x{+}(y{+}y))$
   $=$ $x'=((\bot{+}\bot){+}y) \wedge y'=(x{+}\bot)$
   $=$ $x'=y \wedge y'=x$