148 Let x be an integer variable, and let t be the time variable. Prove the refinement

(a) $x'=1 \ \Leftarrow\ $ **if** $x=1$ **then** ok **else** $x:= div\ x\ 2.\ \ x'=1$ **fi**

(b)√ $R \ \Leftarrow\ $ **if** $x=1$ **then** ok **else** $x:= div\ x\ 2.\ \ t:= t+1.\ \ R$ **fi**

where $R = x'=1 \ \land$ **if** $x\ge 1$ **then** $t' \le t + log\ x$ **else** $t'=\infty$ **fi**

After trying the question, scroll down to the solution.

(a) $x'=1 \;\Leftarrow\; \textbf{if } x=1 \textbf{ then } ok \textbf{ else } x:= div\ x\ 2.\ \ x'=1 \textbf{ fi}$

§ $\textbf{if } x=1 \textbf{ then } ok \textbf{ else } x:= div\ x\ 2.\ \ x'=1 \textbf{ fi}$ — expand ok ; substitution law

$=$ $\textbf{if } x=1 \textbf{ then } x'=x \land t'=t \textbf{ else } x'=1 \textbf{ fi}$ — specialization and monotonicity

$\Rightarrow$ $\textbf{if } x=1 \textbf{ then } x'=x \textbf{ else } x'=1 \textbf{ fi}$ — context

$=$ $\textbf{if } x=1 \textbf{ then } x'=1 \textbf{ else } x'=1 \textbf{ fi}$ — case idempotence

$=$ $x'=1$

(b)√ $R \;\Leftarrow\; \textbf{if } x=1 \textbf{ then } ok \textbf{ else } x:= div\ x\ 2.\ \ t:= t+1.\ \ R \textbf{ fi}$

where $R \;=\; x'=1 \;\land\; \textbf{if } x\ge 1 \textbf{ then } t' \le t + log\ x \textbf{ else } t'=\infty \textbf{ fi}$

§ see book Section 4.2