175 (alternating sum) Write a program to find the alternating sum $L\ 0 - L\ 1 + L\ 2 - L\ 3 + ...$ of a list L of numbers.

After trying the question, scroll down to the solution.

§ Let s be a number variable, and n be a natural variable.

$$s' = (\Sigma i: 0,..\#L\cdot (-1)^i \times L\,i) \ \wedge\ t' = t+\#L \ \Leftarrow$$
$$s{:=}\ 0.\ \ n{:=}\ 0.\ \ s' = s + (\Sigma i: n,..\#L\cdot (-1)^i \times L\,i) \ \wedge\ t' = t+\#L-n$$

Proof: two substitutions.

$$s' = s + (\Sigma i: n,..\#L\cdot (-1)^i \times L\,i) \ \wedge\ t' = t+\#L-n \ \Leftarrow$$
$$\textbf{if } n{=}\#L \textbf{ then } ok$$
$$\textbf{else}\ \ s{:=}\ s + (-1)^n \times L\,n.\ \ n{:=}\ n+1.\ \ t{:=}\ t+1.$$
$$s' = s + (\Sigma i: n,..\#L\cdot (-1)^i \times L\,i) \ \wedge\ t' = t+\#L-n \textbf{ fi}$$

Proof: by cases. First case:

$$s' = s + (\Sigma i: n,..\#L\cdot (-1)^i \times L\,i) \ \wedge\ t' = t+\#L-n \ \Leftarrow\ n{=}\#L \wedge s'{=}s \wedge n'{=}n \wedge t'{=}t$$

because if $n{=}\#L$ then $(\Sigma i: n,..\#L\cdot (-1)^i \times L\,i) = 0$.

Second case, starting with its right side:

$$n{\neq}\#L \wedge (\ \ s{:=}\ s + (-1)^n \times L\,n.\ \ n{:=}\ n+1.\ \ t{:=}\ t+1.$$
$$s' = s + (\Sigma i: n,..\#L\cdot (-1)^i \times L\,i) \ \wedge\ t' = t+\#L-n\ ) \qquad \text{Substitution 3 times}$$
$$= \quad n{\neq}\#L \ \wedge\ s' = s + (-1)^n \times L\,n + (\Sigma i: n+1,..\#L\cdot (-1)^i \times L\,i) \ \wedge\ t' = t+1+\#L-n-1$$
simplify, incorporating $(-1)^n \times L\,n$ into the summation
$$= \quad n{\neq}\#L \ \wedge\ s' = s + (\Sigma i: n,..\#L\cdot (-1)^i \times L\,i) \ \wedge\ t' = t+\#L-n \qquad \text{specialization}$$
$$\Longrightarrow \quad s' = s + (\Sigma i: n,..\#L\cdot (-1)^i \times L\,i) \ \wedge\ t' = t+\#L-n$$

which is the left side. If $s{:=}\ s + (-1)^n \times L\,n$ is program, we're done. If not, we have to refine it. Even if it is program, we might still want to refine it.

$$s{:=}\ s + (-1)^n \times L\,n \ \Leftarrow\ \textbf{if } even\ n \textbf{ then } s{:=}\ s + L\,n \textbf{ else } s{:=}\ s-Ln \textbf{ fi}$$

The proof is easy.