213 (next combination) You are given a sorted list of m different numbers, all in the range $0,..n$. Write a program to find the lexicographically next sorted list of m different numbers, all in the range $0,..n$.

After trying the question, scroll down to the solution.

§ Here is the last sorted list of 5 different numbers all in the range 0,..10 .

[5; 6; 7; 8; 9]

At index i , the largest possible item is $n–m+i$. Strategy: find the last item that is below its maximum, increase it by 1 , then fill up the following items in increasing order. For example, if the sorted list of 5 different numbers in the range 0,..10 is

[2; 4; 7; 8; 9]

the last item that is below its maximum is the 4 . So increase the 4 to 5, then fill up the rest and get

[2; 5; 6; 7; 8]

To find the last, search from the end back toward the beginning. To make the specification implementable, we have to decide what to do if we are given the last list; I choose that we leave it as is.

Let L be a list variable whose initial value is the given sorted list of length m with items all in $0,..n$. Let i be a *nat* variable used to index L . Define specifications

S = **if** $L = [n–m;..n]$ **then** $L'=L$ **else** UNFINISHED **fi**
A = UNFINISHED
B = UNFINISHED

The refinements are

S ⇐ $i:= m.\ A$

A ⇐ **if** $i=0$ **then** *ok* — the given list is the last
else $i:= i–1.$ **if** $L\ i = n–m+i$ **then** A — the item is max
else $L\ i:= L\ i + 1.\ B$ **fi fi**

B ⇐ $i:= i+1.$ **if** $i=m$ **then** *ok* **else** $L\ i:= L(i–1)+1.\ B$ **fi**

The proofs are UNFINISHED