321   Let $W \Longleftarrow$ **while** b **do** P **od** be an alternate notation for $W \Longleftarrow$ **if** b **then** $P. W$ **else** ok **fi** .
      Let $R \Longleftarrow$ **do** P **until** b **od** be an alternate notation for $R \Longleftarrow P.$ **if** b **then** ok **else** R **fi** .
      Now prove
$$(R \Longleftarrow \textbf{do } P \textbf{ until } b \textbf{ od}) \land (W \Longleftarrow \textbf{while } \neg b \textbf{ do } P \textbf{ od})$$
$$\Longleftarrow (R \Longleftarrow P. W) \land (W \Longleftarrow \textbf{if } b \textbf{ then } ok \textbf{ else } R \textbf{ fi})$$

After trying the question, scroll down to the solution.

§ Starting with the bottom side,

$(R \Longleftarrow P.\,W) \wedge (W \Longleftarrow \textbf{if } b \textbf{ then } ok \textbf{ else } R \textbf{ fi})$ idempotent law

$= \quad (R \Longleftarrow P.\,W) \wedge (R \Longleftarrow P.\,W) \wedge (W \Longleftarrow \textbf{if } b \textbf{ then } ok \textbf{ else } R \textbf{ fi})$

In the leftmost conjunct, strengthen W by using the rightmost conjunct.
This strengthens the antecedent of an implication, and so weakens the implication.

$\Longrightarrow \quad (R \Longleftarrow P.\,\textbf{if } b \textbf{ then } ok \textbf{ else } R \textbf{ fi}) \wedge (R \Longleftarrow P.\,W) \wedge (W \Longleftarrow \textbf{if } b \textbf{ then } ok \textbf{ else } R \textbf{ fi})$

In the rightmost conjunct strengthen R by using the middle conjunct.
This strengthens the antecedent of an implication, and so weakens the implication.

$\Longrightarrow \quad (R \Longleftarrow P.\,\textbf{if } b \textbf{ then } ok \textbf{ else } R \textbf{ fi}) \wedge (R \Longleftarrow P.\,W) \wedge (W \Longleftarrow \textbf{if } b \textbf{ then } ok \textbf{ else } P.\,W \textbf{ fi})$

Drop the middle conjunct.

$\Longrightarrow \quad (R \Longleftarrow P.\,\textbf{if } b \textbf{ then } ok \textbf{ else } R \textbf{ fi}) \wedge (W \Longleftarrow \textbf{if } b \textbf{ then } ok \textbf{ else } P.\,W \textbf{ fi})$

use case reversal in the second conjunct

$= \quad (R \Longleftarrow P.\,\textbf{if } b \textbf{ then } ok \textbf{ else } R \textbf{ fi}) \wedge (W \Longleftarrow \textbf{if } \neg b \textbf{ then } P.\,W \textbf{ else } ok \textbf{ fi})$

use alternative notations

$= \quad (R \Longleftarrow \textbf{do } P \textbf{ until } b \textbf{ od}) \wedge (W \Longleftarrow \textbf{while } \neg b \textbf{ do } P \textbf{ od})$