325 Here is a nest of loops. All **exit**s are shown. What refinements need to be proven in order to prove that this nest of loops refines specification S ?

(a)√

```
do A.
    do B.
        exit 2 when c.
        D. od.
    E od
```

(b)√

```
do A.
    exit 1 when b.
    C.
    do D.
        exit 2 when e.
        F.
        exit 1 when g.
        H od.
    I od
```

(c)

```
do A.
    do B.
        do C.
            exit 1 when u.
            exit 2 when v.
            exit 3 when w.
            D od.
        E od.
    F od
```

(d)

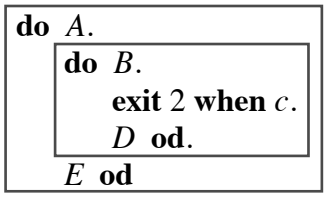
```
do A.
    do B.
        exit 2 when u.
        C.
        exit 1 when v.
        D od.
    E.
    exit 1 when w.
    F.
    do G.
        do H.
            exit 1 when x.
            I.
            exit 2 when y.
            J od.
        K.
        exit 1 when z.
        L od.
    M od
```

(e)

```
do A.
    exit 1 when u.
    do B.
        exit 2 when v.
        exit 1 when ⊤ od od
```

After trying the question, scroll down to the solution.

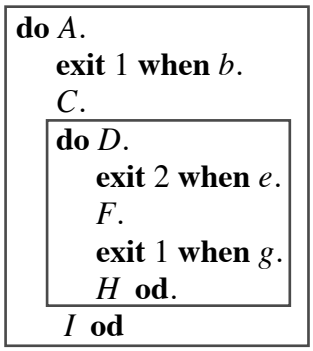
(a)√



§ Using R as specification for the inner loop,

$S \Leftarrow A.\ R$
$R \Leftarrow B.$ **if** c **then** ok **else** $D.\ R$ **fi**

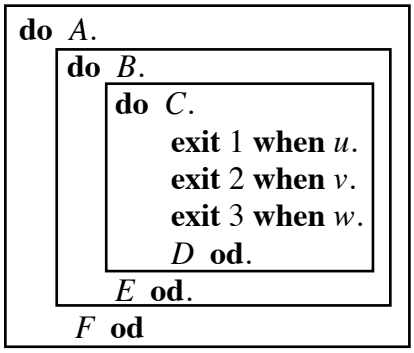
(b)√



§ Using R as specification for the inner loop,

$S \Leftarrow A.$ **if** b **then** ok **else** $C.\ R$ **fi**
$R \Leftarrow D.$ **if** e **then** ok **else** $F.$ **if** g **then** $I.\ S$ **else** $H.\ R$ **fi fi**

(c)

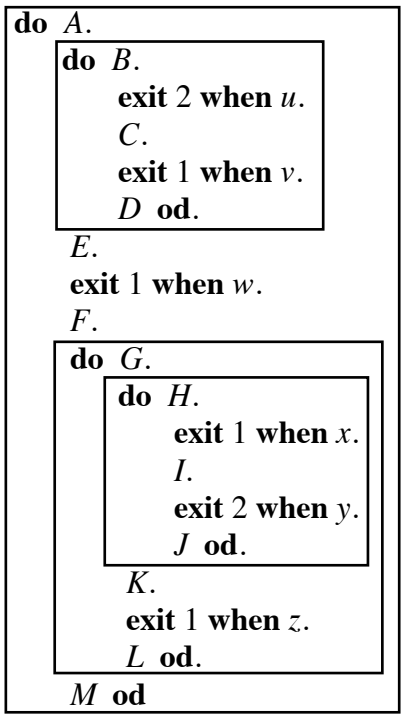


§ Using R and Q as specifications for the inner loops,

$S \Leftarrow A.\ R$
$R \Leftarrow B.\ Q$
$Q \Leftarrow C.$ **if** u **then** $E.\ R$
else if v **then** $F.\ S$
else if w **then** ok
else $D.\ Q$ **fi fi fi**

Specification R isn't necessary; its two uses could be replaced by $B.Q$.

(d)

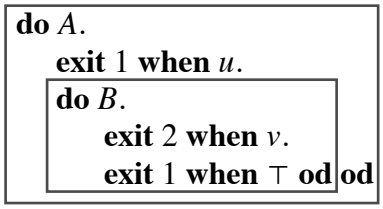


§ Using P , Q , and R as specifications for the inner loops,

$S \Leftarrow A.\ P$
$P \Leftarrow B.$ **if** u **then** ok
else $C.$ **if** v **then** $E.$ **if** w **then** ok
else $F.\ Q$ **fi**
else $D.\ P$ **fi fi**
$Q \Leftarrow G.\ R$
$R \Leftarrow H.$ **if** x **then** $K.$ **if** z **then** $M.\ S$
else $L.\ Q$ **fi**
else $I.$ **if** y **then** $M.\ S$
else $J.\ R$ **fi fi**

Specification Q isn't necessary; its two uses could be replaced by $G.R$.

(e)



§ Using R as specification for the inner loop,

$S \Leftarrow A.$ **if** u **then** ok **else** R **fi**
$R \Leftarrow B.$ **if** v **then** ok **else** S **fi**

or we can combine these two refinements into one, not requiring any new specification:

$S \Leftarrow A.$ **if** u **then** ok **else** $B.$ **if** v **then** ok **else** S **fi fi**