327 (square) Let n be natural and let s be a natural variable. Using a **for**-loop, without using multiplication or exponentiation, write a program for $s'=n^2$.

After trying the question, scroll down to the solution.

§ A'll use the invariant form of **for**-loop.

$s'=n^2 \Leftarrow s:=0.\ A\,0 \Rightarrow A'n$

$A\,0 \Rightarrow A'n \Leftarrow$ **for** $k:=0;..n$ **do** $k: 0,..n \wedge A\,k \Rightarrow A'(k+1)$ **od**

$k: 0,..n \wedge A\,k \Rightarrow A'(k+1) \Leftarrow s:= s +$ something

To complete the final refinement, we need to define $A\,k$. Here's one way.

$A\,k \;=\; s = k \times n$

Now to change $k \times n$ into $(k+1) \times n$ we need to add

$(k+1) \times n - k \times n \;=\; n$

So we complete the solution:

$k: 0,..n \wedge A\,k \Rightarrow A'(k+1) \Leftarrow s:= s+n$

Here's another way to define $A\,k$.

$A\,k \;=\; s = k^2$

Now to change k^2 into $(k+1)^2$ we need to add

$(k+1)^2 - k^2 \;=\; k+k+1$

So we complete the solution:

$k: 0,..n \wedge A\,k \Rightarrow A'(k+1) \Leftarrow s:= s+k+k+1$