343 (three cards) There are three cards. One is red on both sides; one is white on both sides; one is red on one side and white on the other side. You are looking at one side of one card, and it is red. What is the probability that its other side is also red?

After trying the question, scroll down to the solution.

§ Well, we're not looking at the card that's white on both sides. So we're looking at one of the other two cards. So the probability that the other side is red is 1/2 . Right? Wrong!

Let the cards be $R$ (both sides red), $W$ (both sides white), and $M$ (mixed). Each card has a side 0 and a side 1 . Define function *color* as follows.

$color\ R\ 0 = red$
$color\ R\ 1 = red$
$color\ W\ 0 = white$
$color\ W\ 1 = white$
$color\ M\ 0 = red$
$color\ M\ 1 = white$

Let $c$ be the card we are looking at, and let $s$ be the side of card $c$ that we are looking at. We see *red* , and we express that as

$color\ c'\ s' = red$

But a binary expression is not necessarily a distribution. To create a proportional distribution, we must divide by the sum.

$(color\ c'\ s' = red) / (\Sigma c, s\cdot\ color\ c\ s = red)$ look at the definition of *color*
$= (color\ c'\ s' = red) / (\ 1+1+0+0+1+0)$
$= (color\ c'\ s' = red) / 3$

We want to know whether the other side of the same card is *red* . The other side from $s$ is side $1–s$ . So that's

$color\ c\ (1–s) = red$

Now we calculate. We see red; is the other side red?

$(color\ c'\ s' = red) / 3.\ \ color\ c\ (1–s) = red$

$= \Sigma c'', s''\cdot\ (color\ c''\ s'' = red) / 3 \times (color\ c''\ (1–s'') = red)$

$= (color\ R\ 0 = red) / 3 \times (color\ R\ 1 = red)$
$+ (color\ R\ 1 = red) / 3 \times (color\ R\ 0 = red)$
$+ (color\ W\ 0 = red) / 3 \times (color\ W\ 1 = red)$
$+ (color\ W\ 1 = red) / 3 \times (color\ W\ 0 = red)$
$+ (color\ M\ 0 = red) / 3 \times (color\ M\ 1 = red)$
$+ (color\ M\ 1 = red) / 3 \times (color\ M\ 0 = red)$

$= 1 / 3 \times 1$
$+ 1 / 3 \times 1$
$+ 0 / 3 \times 0$
$+ 0 / 3 \times 0$
$+ 1 / 3 \times 0$
$+ 0 / 3 \times 1$

$= 2/3$

The probability that the other side of the card I'm looking at is also red is 2/3 .