363 (standard deviation) Let P be any distribution of final states (primed variables), and let e be any number expression or binary expression over initial states (unprimed variables). The standard deviation of e according to distribution P is

$$((P.\ e^2) - (P.\ e)^2)^{1/2}$$

Standard deviation measures the margin of error.

(a) Prove standard deviation is equivalent to

$$(P.\ (e-(P.\ e))^2)^{1/2}$$

(b) Let n vary over $nat+1$. What is the standard deviation of 2 according to distribution $2^{-n'}$?

(c) Let n vary over $nat+1$. What is the standard deviation of 2^{-n} according to distribution $2^{-n'}$?

After trying the question, scroll down to the solution.

(a) Prove standard deviation is equivalent to

$$(P.\ (e-(P.\ e))^2)^{1/2}$$

(b) Let n vary over $nat+1$. What is the standard deviation of 2 according to distribution $2^{-n'}$?

§ $((2^{-n'}.\ 2^2) - (2^{-n'}.\ 2)^2)^{1/2}$ — definition of . twice

$= ((\Sigma n'': nat+1\cdot\ 2^{-n''} \times 2^2) - (\Sigma n'': nat+1\cdot\ 2^{-n''} \times 2)^2)^{1/2}$ — drop the primes

$= ((\Sigma n: nat+1\cdot\ 2^{-n} \times 2^2) - (\Sigma n: nat+1\cdot\ 2^{-n} \times 2)^2)^{1/2}$ — factor twice

$= ((\Sigma n: nat+1\cdot\ 2^{-n}) \times 2^2 - ((\Sigma n: nat+1\cdot\ 2^{-n}) \times 2)^2)^{1/2}$ — sum twice

$= (1 \times 2^2 - (1 \times 2)^2)^{1/2}$

$= (4 - 4)^{1/2}$

$= 0$

The number 2 is a constant; it does not vary.

(c) Let n vary over $nat+1$. What is the standard deviation of 2^{-n} according to distribution $2^{-n'}$?

§ $((2^{-n'}.\ (2^{-n})^2) - (2^{-n'}.\ 2^{-n})^2)^{1/2}$ — definition of . twice

$= ((\Sigma n'': nat+1\cdot\ 2^{-n''} \times (2^{-n''})^2) - (\Sigma n'': nat+1\cdot\ 2^{-n''} \times 2^{-n''})^2)^{1/2}$ — drop the primes

$= ((\Sigma n: nat+1\cdot\ 2^{-n} \times (2^{-n})^2) - (\Sigma n: nat+1\cdot\ 2^{-n} \times 2^{-n})^2)^{1/2}$ — arithmetic

$= ((\Sigma n: nat+1\cdot\ 2^{-3\times n}) - (\Sigma n: nat+1\cdot\ 2^{-2\times n})^2)^{1/2}$ — sum twice

$= ((1/7) - (1/3)^2)^{1/2}$

$= ((1/7) - (1/9))^{1/2}$

$= (2/63)^{1/2}$

$::$ 0.178 approximately

As n varies, 2^{-n} varies. The probability of each value of n is 2^{-n} , and the standard deviation (variability) of 2^{-n} is approximately 0.178 .