371 Here is a possible alternative construction axiom for *nat* .

$$0, 1, nat+nat: nat$$

(a) What induction axiom goes with it?

(b) Are the construction axiom given and your induction axiom of part (a) satisfactory as a definition of *nat*?

After trying the question, scroll down to the solution.

(a) What induction axiom goes with it?

§ $\quad 0, 1, B+B: B \;\Rightarrow\; nat: B$

(b) Are the construction axiom given and your induction axiom of part (a) satisfactory as a definition of *nat*?

§ Yes. To prove they are sufficient to define *nat*, use them to prove ordinary *nat* construction and induction. So, assume the new construction and induction (and do not assume anything else about *nat*). Now prove ordinary *nat* construction.

$0, nat+1: nat$ UNFINISHED

$= \quad \top$

Now prove ordinary *nat* induction.

$0, B+1: B \;\Rightarrow\; nat: B$ UNFINISHED

$= \quad \top$

We don't really need to prove they are necessary to define *nat*, but if we want to, assume ordinary *nat* construction and induction (and do not assume anything else about *nat*). Now prove the new construction.

$0, 1, nat+nat: nat$ from ordinary construction, we have both $0: nat$ and $1: nat$

$= \quad nat+nat: nat$ bunch-element conversion law

$= \quad \forall n: nat+nat\cdot \exists m: nat\cdot n=m$ UNFINISHED

$= \quad \top$

Now prove the new induction.

$0, 1, B+B: B \;\Rightarrow\; nat: B$ UNFINISHED

$= \quad \top$