

391 For each of the following fixed-point equations, what does recursive construction yield?  
Does it satisfy the fixed-point equation?

(a)  $P = \lambda n: \text{nat}. n=0 \wedge P=\text{null} \vee n: P+1$

(b)  $Q = \lambda x: \text{xnat}. x=0 \wedge Q=\text{null} \vee x: Q+1$

After trying the question, scroll down to the solution.

(a)  $P = \S n: \text{nat} \cdot n=0 \wedge P=\text{null} \vee n: P+1$

§  $P_0 = \text{null}$

$P_{n+1} = n$

$P_\infty = \infty$  which does not satisfy the fixed-point equation.

§ $x: \Downarrow n \cdot x: P_n = \text{null}$  which does not satisfy the fixed-point equation either.

Maybe there isn't any solution.

(b)  $Q = \S x: \text{nat} \cdot x=0 \wedge Q=\text{null} \vee x: Q+1$

§  $Q_0 = \text{null}$

$Q_{n+1} = n$

$Q_\infty = \infty$  which does satisfy the fixed-point equation.

§ $x: \Downarrow n \cdot x: Q_n = \text{null}$  which does not satisfy the fixed-point equation.