

395 Let *truer* be a bunch of strings of binary values defined by the construction and induction axioms

$nil, \perp; truer, truer; \top : truer$

$nil, \perp; B, B; \top : truer \Rightarrow truer: B$

Given a string of binary values, write a program to determine if the string is in *truer* .

After trying the question, scroll down to the solution.

§

$$truer = * \perp; * \top$$

A string in *truer* is a string of \perp 's of any length followed by a string of \top 's of any length. In other words, it is a monotonic string.

Let s be a string of binary values. Let b be a binary variable whose final value will tell us if s is *truer*. The specification is

$$P = b' = (s: * \perp; * \top)$$

The program can be $b := s: * \perp; * \top$ if $:$ and $*$ are implemented. I suppose they are not implemented. We need a natural variable n to index s , and two more specifications.

$$Q = b' = (s_{n.. \leftrightarrow s}: * \perp; * \top)$$

$$R = b' = (s_{n.. \leftrightarrow s}: * \top)$$

Here are the refinements.

$$P \Leftarrow n := 0. Q$$

$$Q \Leftarrow \text{if } n = \leftrightarrow s \text{ then } b := \top \\ \text{else if } s_n \text{ then } n := n + 1. R \\ \text{else } n := n + 1. Q \text{ fi fi}$$

$$R \Leftarrow \text{if } n = \leftrightarrow s \text{ then } b := \top \\ \text{else if } s_n \text{ then } n := n + 1. R \\ \text{else } b := \perp \text{ fi fi}$$