403 Section 6.1 defines program *zap* by the fixed-point equation

$$zap \;=\; \textbf{if } x{=}0 \textbf{ then } y{:=}\,0 \textbf{ else } x{:=}\,x{-}1.\;\; t{:=}\,t{+}1.\;\; zap \textbf{ fi}$$

(a)√ What axiom is needed to make *zap* the weakest fixed-point?

(b) What axiom is needed to make *zap* the strongest fixed-point?

(c) Section 6.1 gives six solutions to this equation. Find more solutions. Hint: strange things can happen at time ∞.

After trying the question, scroll down to the solution.

(a)√ What axiom is needed to make *zap* the weakest fixed-point?

§ $(\forall \sigma, \sigma' \cdot\ Z\ =$ **if** $x{=}0$ **then** $y{:=}\ 0$ **else** $x{:=}\ x{-}1.\ \ t{:=}\ t{+}1.\ \ Z$ **fi**$)\ \Rightarrow\ (\forall \sigma, \sigma' \cdot\ zap \Leftarrow Z)$

(b) What axiom is needed to make *zap* the strongest fixed-point?

§ $(\forall \sigma, \sigma' \cdot\ Z\ =$ **if** $x{=}0$ **then** $y{:=}\ 0$ **else** $x{:=}\ x{-}1.\ \ t{:=}\ t{+}1.\ \ Z$ **fi**$)\ \Rightarrow\ (\forall \sigma, \sigma' \cdot\ Z \Leftarrow zap)$

(c) Section 6.1 gives six solutions to this equation. Find more solutions. Hint: strange things can happen at time ∞.

§ **if** $x{\geq}0$ **then** $x'{=}y'{=}0 \wedge\ t' = t{+}x$ **else** $x'{=}y'{=}17 \wedge\ t'{=}\infty$ **fi**