405 Let x and y be rational variables. Define program *zot* by the fixed-point equation

$$zot \;=\; \textbf{if } x{=}y \textbf{ then } y{:=}\,0 \textbf{ else } x{:=}\,(x{+}y)/2.\;\; zot \textbf{ fi}$$

(a) Add recursive time.

(b) Give two solutions to this equation (with recursive time added) (considering *zot* as the unknown). (No proof needed.)

(c) The definition of *zot* makes it a solution (fixed-point) of an equation. What axiom is needed to make *zot* the weakest solution (weakest fixed-point)?

After trying the question, scroll down to the solution.

(a) Add recursive time.

§ $zot\ \ =\ \ \textbf{if}\ x{=}y\ \textbf{then}\ y{:=}\ 0\ \textbf{else}\ x{:=}\ (x{+}y)/2.\ \ t{:=}\ t{+}1.\ \ zot\ \textbf{fi}$

(b) Give two solutions to this equation (with recursive time added) (considering zot as the unknown). (No proof needed.)

§ Here are three solutions. The first is the result of recursive construction if we start with $\top$.

$$x{=}y \Rightarrow (y{:=}\ 0)$$

The next is the result of recursive construction if we start with $t'{\ge}t$.

$$\textbf{if}\ x{=}y\ \textbf{then}\ y{:=}0\ \textbf{else}\ t'{=}\infty\ \textbf{fi}$$

If execution starts with $x{\ne}y$, it's an infinite loop, so we can say anything about the final values x' and y' , since they are unobservable. The next solution is

$$\textbf{if}\ x{=}y\ \textbf{then}\ y{:=}0\ \textbf{else}\ x'{=}12 \land y'{=}17 \land t'{=}\infty\ \textbf{fi}$$

(c) The definition of zot makes it a solution (fixed-point) of an equation. What axiom is needed to make zot the weakest solution (weakest fixed-point)?

§
$$(\forall x, y, t, x', y', t' \cdot\ Z\ =\ \textbf{if}\ x{=}0\ \textbf{then}\ y{:=}\ 0\ \textbf{else}\ x{:=}\ (x{+}y)/2.\ \ t{:=}\ t{+}1.\ \ Z\ \textbf{fi})$$
$$\Rightarrow\ (\forall x, y, t, x', y', t' \cdot Z \Rightarrow zot)$$