410 Let $x$ be an integer variable. Using the recursive time measure, add time and then find the strongest implementable specifications $P$ and $Q$ that you can find for which

$$P \Leftarrow x' \geq 0.\ Q$$

$$Q \Leftarrow \textbf{if } x=0 \textbf{ then } ok \textbf{ else } x:=x-1.\ Q \textbf{ fi}$$

Assume that $x' \geq 0$ takes no time.

After trying the question, scroll down to the solution.

§ Adding time, we have

$$P \Leftarrow x'\ge 0 \wedge t'=t.\ \ Q$$

$$Q \Leftarrow \textbf{if } x=0 \textbf{ then } ok \textbf{ else } x:= x-1.\ \ t:= t+1.\ Q \textbf{ fi}$$

For $P$ there is a unique strongest implementable specification; for $Q$ there are many.

$$P = x'=0 \wedge t'\ge t$$

$$Q = \textbf{if } x\ge 0 \textbf{ then } x'=0 \wedge t' = t+x \textbf{ else } x'=17 \wedge t'=\infty \textbf{ fi}$$

or choose any number in place of 17 .

$$
\begin{aligned}
& \textbf{if } x=0 \textbf{ then } ok \textbf{ else } x:= x-1.\ \ t:= t+1.\ Q \textbf{ fi} \\
= \ & \textbf{if } x=0 \textbf{ then } ok \\
& \textbf{else } x:= x-1.\ \ t:= t+1.\ \ \textbf{if } x\ge 0 \textbf{ then } x'=0 \wedge t' = t+x \textbf{ else } x'=17 \wedge t'=\infty \textbf{ fi fi} \\
& \hspace{20em} \text{substitution law twice} \\
= \ & \textbf{if } x=0 \textbf{ then } ok \textbf{ else if } x-1\ge 0 \textbf{ then } x'=0 \wedge t' = t+1+x-1 \textbf{ else } x'=17 \wedge t'=\infty \textbf{ fi fi} \\
= \ & \textbf{if } x=0 \textbf{ then } ok \textbf{ else if } x\ge 1 \textbf{ then } x'=0 \wedge t' = t+x \textbf{ else } x'=17 \wedge t'=\infty \textbf{ fi fi} \\
= \ & \textbf{if } x=0 \textbf{ then } x'=0 \wedge t' = t+x \textbf{ else if } x\ge 1 \textbf{ then } x'=0 \wedge t' = t+x \textbf{ else } x'=17 \wedge t'=\infty \textbf{ fi fi} \\
= \ & \textbf{if } x\ge 0 \textbf{ then } x'=0 \wedge t' = t+x \textbf{ else } x'=17 \wedge t'=\infty \textbf{ fi} \\
= \ & Q
\end{aligned}
$$

$$
\begin{aligned}
& x'\ge 0 \wedge t'=t.\ \ Q \\
= \ & x'\ge 0 \wedge t'=t.\ \ \textbf{if } x\ge 0 \textbf{ then } x'=0 \wedge t' = t+x \textbf{ else } x'=17 \wedge t'=\infty \textbf{ fi} \\
= \ & \exists x'', t''\cdot\ \ x''\ge 0 \wedge t''=t \wedge \textbf{if } x''\ge 0 \textbf{ then } x'=0 \wedge t' = t''+x'' \textbf{ else } x'=17 \wedge t'=\infty \textbf{ fi} \\
= \ & \exists x''\cdot\ \ x''\ge 0 \wedge \textbf{if } x''\ge 0 \textbf{ then } x'=0 \wedge t' = t+x'' \textbf{ else } x'=17 \wedge t'=\infty \textbf{ fi} \\
= \ & \exists x''\cdot\ \ x''\ge 0 \wedge x'=0 \wedge t' = t+x'' \\
= \ & x'=0 \wedge t'\ge t \\
= \ & P
\end{aligned}
$$