43 We defined bunch *null* with the axiom $null: A$. Is there any harm in defining bunch *all* with the axiom $A: all$?

After trying the question, scroll down to the solution.

§ With just Binary Theory, Number Theory, Character Theory, and Bunch Theory, there is no harm (inconsistency) in defining all with the axiom $A: all$. Even when we add Set Theory (in this book; we don't yet have set comprehension) there is no harm. But when we add Function Theory, specifically the § quantifier, we have an inconsistency known as "Russell's Paradox". Let

$$R = \{s: ⁄all\cdot \neg\ s{\in}s\}$$

Then R is the set of all sets that are not members of themselves. Or, without abbreviation,

$$R = \{§\langle s: ⁄all \rightarrow \neg\ s{\in}s\rangle\}$$

Then

	$R{\in}R$	definition of R
$=$	$R \in \{s: ⁄all\cdot \neg\ s{\in}s\}$	$\in$ axiom
$=$	R: $§s: ⁄all\cdot \neg\ s{\in}s$	solution law
$=$	$R: ⁄all \wedge \neg\ R{\in}R$	definition of R
$=$	$\{s: ⁄all\cdot \neg\ s{\in}s\}: ⁄all \wedge \neg\ R{\in}R$	$⁄$ axiom
$=$	$(§s: ⁄all\cdot \neg\ s{\in}s): all \wedge \neg\ R{\in}R$	all axiom
$=$	$\top \wedge \neg\ R{\in}R$	identity law
$=$	$\neg\ R{\in}R$	

and we have inconsistency.

It might be nice to have all , and to weaken the solution law to accommodate it. But I have stayed with standard mathematics, excluding all and including the strong form of solution law.

Even without all , we still have a benign form of Russell's Paradox (Exercise 48); it is not an inconsistency, but it may disturb some people.