473 Implementer's variables p, q: *real* represent two points along a line. Each number tells the distance of one point from the origin (a standard point). They must be reimplemented by one implementer's variable r: *real* which tells the distance from p to q. For examples, if $p=3$ and $q=5$, then $r=2$; if $p=5$ and $q=3$, then $r=-2$.

(a) What is the data transformer?

(b) A user has binary variable b and operation

$$compare \;=\; b:= q\ge p$$

Use your transformer from part (a) to transform operation *compare*.

After trying the question, scroll down to the solution.

(a) What is the data transformer?

§ $r = q{-}p$

(b) A user has binary variable b and operation

$compare \;=\; b{:=}\, q{\ge}p$

Use your transformer from part (a) to transform operation $compare$.

§ $\forall p, q\cdot\ \ r = q{-}p \ \Rightarrow\ \exists p', q'\cdot\ \ r' = q'{-}p' \wedge (b{:=}\, q{\ge}p)$ expand assignment

$=\ \forall p, q\cdot\ \ r = q{-}p \ \Rightarrow\ \exists p', q'\cdot\ \ r' = q'{-}p' \ \wedge\ b'{=}(q{\ge}p) \ \wedge\ p'{=}p \ \wedge\ q'{=}q$ one-point twice

$=\ \forall p, q\cdot\ \ r = q{-}p \ \Rightarrow\ r' = q{-}p \ \wedge\ b'{=}(q{\ge}p)$ context

$=\ \forall p, q\cdot\ \ r = q{-}p \ \Rightarrow\ r'{=}r \ \wedge\ b'{=}(r{\ge}0)$ some law of arithmetic

$=\ \forall p, q\cdot\ \ p = q{-}r \ \Rightarrow\ r'{=}r \ \wedge\ b'{=}(r{\ge}0)$ one-point and idempotent

$=\ r'{=}r \ \wedge\ b'{=}(r{\ge}0)$ definition of assignment

$=\ b{:=}\, r{\ge}0$