

5 Simplify each of the following binary expressions.

(a)  $x \wedge \neg x$

(b)  $x \vee \neg x$

(c)  $x \Rightarrow \neg x$

(d)  $x \Leftarrow \neg x$

(e)  $x = \neg x$

(f)  $x \neq \neg x$

After trying the question, scroll down to the solution.

Solutions

§(a)	$x \wedge \neg x$	double negation
=	$\neg \neg(x \wedge \neg x)$	noncontradiction
=	$\neg \top$	binary law
=	$\neg \neg \perp$	double negation
=	$\perp$	
§(b)	$x \vee \neg x$	law of excluded middle
=	$\top$	
§(c)	$x \Rightarrow \neg x$	double negation
=	$\neg \neg x \Rightarrow \neg x$	indirect proof
=	$\neg x$	
§(d)	$x \Leftarrow \neg x$	indirect proof
=	$x$	
§(e)	$x = \neg x$	exclusion
=	$x \neq x$	unequality
=	$\neg(x = x)$	reflexivity
=	$\neg \top$	binary law
=	$\neg \neg \perp$	double negation
=	$\perp$	
§(f)	$x \neq \neg x$	exclusion
=	$x = \neg \neg x$	double negation
=	$x = x$	reflexive
=	$\top$	