

511 (*T*-strings) Let us call a string  $S$ : \*("a", "b", "c") a *T*-string if no two adjacent nonempty segments are identical:

$$\neg \exists i, j, k. 0 \leq i < j < k \leq \text{len}(S) \wedge S_{i..j} = S_{j..k}$$

Write a program to output all *T*-strings in alphabetical order. (The mathematician Axel Thue proved that there are infinitely many *T*-strings.)

After trying the question, scroll down to the solution.

§ Define  $R =$  (print all  $T$ -strings in alphabetical order) .  
 Define  $Z =$  (print all  $T$ -strings from  $S$  on in alphabetical order) .  
 Define  $T =$  ( $S$  is a  $T$ -string)  $= \neg \exists i, j, k. 0 \leq i < j < k \leq \#S \wedge S_{i..j} = S_{j..k}$  .  
 Define  $U =$  ( $S$  has no adjacent nonempty identical segments of length  $< l$ )  
 $= \neg \exists i, j, k. 0 \leq i < j < k \leq \#S \wedge j - i < l \wedge S_{i..j} = S_{j..k}$  .  
 $R \Leftarrow S := ""$  .  $T \Rightarrow Z$   
 $T \Rightarrow Z \Leftarrow !S. S := S; "a"$  .  $Z$   
 $Z \Leftarrow l := 1$  .  $U \Rightarrow Z$   
 $U \Rightarrow Z \Leftarrow$

**if**  $\#S \geq 2 \times l$   
**then** **if**  $S_{\leftrightarrow S - 2 \times l; \dots \leftrightarrow S - l} = S_{\leftrightarrow S - l; \dots \leftrightarrow S}$   
**then**  $S :=$  (the alphabetically next text that is not longer) .  $Z$   
**else**  $l := l + 1$  .  $U \Rightarrow Z$  **fi**  
**else**  $T \Rightarrow Z$  **fi**

$S :=$  (the alphabetically next text that is not longer)  $\Leftarrow$

**if**  $S_{\leftrightarrow S - 1} = "a"$  **then**  $S := S_{0; \dots \leftrightarrow S - 1}; "b"$

**else if**  $S_{\leftrightarrow S - 1} = "b"$  **then**  $S := S_{0; \dots \leftrightarrow S - 1}; "c"$

**else**  $S := S_{0; \dots \leftrightarrow S - 1}$  .  $S :=$  (the alphabetically next text that is not longer) **fi fi**

The one insight is the fact that a non- $T$ -string cannot be made into a  $T$ -string by extending it, hence the assignment  $S :=$  (the alphabetically next text that is not longer) . We are assured that there is one by Thue.