64 Simplify (no proof)

(a) 0→1 | 1→2 | 2→3 | 3→4 | 4→5 | [0;..5]

(b) [0][0][0][0]

(c) ((3;2)→[10;..15] | 3→[5;..10] | [0;..5]) 3

(d) ([0;..5] [3; 4]) 1

(e) (2;2)→"j" | [["abc"]; ["de"]; ["fghi"]]

(f) #[*nat*]

(g) #[*3]

(h) [3; 4]: [3*4**int*]

(i) [3; 4]: [3; *int*]

(j) [3, 4; 5]: [2**int*]

(k) [(3, 4); 5]: [2**int*]

(l) [3; (4, 5); 6; (7, 8, 9)] ‘ [3; 4; (5, 6); (7, 8)]

After trying the question, scroll down to the solution.

(a) $0\to1 \mid 1\to2 \mid 2\to3 \mid 3\to4 \mid 4\to5 \mid [0;..5]$

§ $0\to1 \mid 1\to2 \mid 2\to3 \mid 3\to4 \mid 4\to5 \mid [0;\ 1;\ 2;\ 3;\ 4]$

$= \quad 0\to1 \mid 1\to2 \mid 2\to3 \mid 3\to4 \mid [0;\ 1;\ 2;\ 3;\ 5]$

$= \quad 0\to1 \mid 1\to2 \mid 2\to3 \mid [0;\ 1;\ 2;\ 4;\ 5]$

$= \quad 0\to1 \mid 1\to2 \mid [0;\ 1;\ 3;\ 4;\ 5]$

$= \quad 0\to1 \mid [0;\ 2;\ 3;\ 4;\ 5]$

$= \quad [1;\ 2;\ 3;\ 4;\ 5]$

$= \quad [1;..6]$

(b) $[0][0][0][0]$

§ $[0][0][0][0]$

$= \quad ([0][0])[0][0]$

$= \quad [0][0][0]$

$= \quad ([0][0])[0]$

$= \quad [0][0]$

$= \quad [0]$

(c) $((3;2)\to[10;..15] \mid 3\to[5;..10] \mid [0;..5])\ 3$

§ $((3;2)\to[10;..15] \mid 3\to[5;..10] \mid [0;..5])\ 3$

$= \quad ((3;2)\to[10;..15] \mid 3\to[5;..10] \mid [0;\ 1;\ 2;\ 3;\ 4])\ 3$

$= \quad ((3;2)\to[10;..15] \mid [0;\ 1;\ 2;\ [5;..10];\ 4])\ 3$

$= \quad ((3;2)\to[10;..15] \mid [0;\ 1;\ 2;\ [5;\ 6;\ 7;\ 8;\ 9];\ 4])\ 3$

$= \quad [0;\ 1;\ 2;\ [5;\ 6;\ [10;..15];\ 8;\ 9];\ 4]\ 3$

$= \quad [5;\ 6;\ [10;..15];\ 8;\ 9]$

(d) $([0;..5]\ [3;\ 4])\ 1$

§ One way:

$([0;..5]\ [3;\ 4])\ 1$

$= \quad [[0;..5]\ 3;\ \ [0;..5]\ 4]\ 1$

$= \quad [0;..5]\ 4$

$= \quad 4$

Another way:

$([0;..5]\ [3;\ 4])\ 1$

$= \quad [0;..5]\ ([3;\ 4]\ 1)$

$= \quad [0;..5]\ 4$

$= \quad 4$

(e) (2;2)→“j” | [[“abc”]; [“de”]; [“fghi”]]

§ Item 2 of [[“abc”]; [“de”]; [“fghi”]] is [“fghi”] and its item 2 is “h” so replacing item 2;2 or [[“abc”]; [“de”]; [“fghi”]] with “j” gives

[[“abc”]; [“de”]; [“fgji”]]

(f) $\#[nat]$

§ 1 because “A nonempty bunch of items is also an item.” page 17

or, informally

$\#[nat]$

$= \quad \#[0, 1, 2, 3, ...]$

$= \quad \#([0], [1], [2], [3], ...)$

$= \quad \#[0], \#[1], \#[2], \#[3], ...$

$= \quad 1, 1, 1, 1, ...$

$= \quad 1$

This is the sort of “proof” that mathematicians accept, but it's not a formal proof because the three dots mean “guess what goes here”. Anyway, the question did not ask for proof.

(g) #[*3]

§ #[*3]

= #[*nil*, 3, 3;3, 3;3;3, ...]

= #([*nil*], [3], [3;3], [3;3;3], ...)

= #[*nil*], #[3], #[3;3], #[3;3;3], ...

= 0, 1, 2, 3, ...

= *nat*

Again, an informal “proof”, but the question did not ask for proof.

(h) [3; 4]: [3*4**int*]

§ 4**int* = *int*; *int*; *int*; *int*

3*4**int* = *int*; *int*; *int*; *int*; *int*; *int*; *int*; *int*; *int*; *int*; *int*; *int*

[3*4**int*] = [*int*; *int*; *int*; *int*; *int*; *int*; *int*; *int*; *int*; *int*; *int*; *int*]

which is all lists of 12 integers, and [3; 4] is not a list of 12 integers, so the answer is

$\bot$

(i) [3; 4]: [3; *int*]

§ [3; *int*] includes all lists of length 2 whose item 0 is 3 and whose item 1 is in *int* .
The list [3; 4] is one of them, so the answer is

$\top$

(j) [3, 4; 5]: [2**int*]

§ [2**int*] includes all lists of length 2 both of whose items are in *int* .

[3, 4; 5]

= [3, (4; 5)]

= [3], [4; 5]

and [3] is not a list of length 2 , so the answer is

$\bot$

(k) [(3, 4); 5]: [2**int*]

§ [(3, 4); 5] = [3;5, 4;5] = [3; 5], [4; 5]

and both these lists are of length 2 and both items of each are in *int* so the answer is

$\top$

(l) [3; (4, 5); 6; (7, 8, 9)] ‘ [3; 4; (5, 6); (7, 8)]

§ [3; (4, 5); 6; (7, 8, 9)] ‘ [3; 4; (5, 6); (7, 8)]

= ([3; 4; 6; 7], [3; 5; 6; 7], [3; 4; 6; 8], [3; 5; 6; 8], [3; 4; 6; 9], [3; 5; 6; 9])
‘ ([3; 4; 5; 7], [3; 4; 6; 7], [3; 4; 5; 8], [3; 4; 6; 8])

= [3; 4; 6; 7], [3; 4; 6; 8]

= [3; 4; 6; (7, 8)]