

69

In each of the following, replace p by

$$\langle x: int \rightarrow \langle y: int \rightarrow \langle z: int \rightarrow x \geq 0 \wedge x^2 \leq y \wedge \forall z: int. z^2 \leq y \Rightarrow z \leq x \rangle \rangle \rangle$$

and simplify, assuming $x, y, z, u, w: int$.

(a)

§

$$p(x+y)(2xu+w)z$$

$$\langle x: int \rightarrow \langle y: int \rightarrow \langle z: int \rightarrow x \geq 0 \wedge x^2 \leq y \wedge \forall z: int. z^2 \leq y \Rightarrow z \leq x \rangle \rangle \rangle (x+y)(2xu+w)z$$

Variables x , y , and z appear both locally and nonlocally.Variable z is introduced twice locally.To avoid confusion, I will rename the local variables to a , b , c , and d .(Since the first local z is unused, I don't need to rename it to c , but I will anyway.)

$$= \langle a: int \rightarrow \langle b: int \rightarrow \langle c: int \rightarrow a \geq 0 \wedge a^2 \leq b \wedge \forall d: int. d^2 \leq b \Rightarrow d \leq a \rangle \rangle \rangle (x+y)(2xu+w)z$$

apply 3 times

$$= x+y \geq 0 \wedge (x+y)^2 \leq 2xu+w \wedge \forall d: int. d^2 \leq 2xu+w \Rightarrow d \leq x+y$$

(b)

§

$$p(x+y)(2xu+w)$$

$$\langle x: int \rightarrow \langle y: int \rightarrow \langle z: int \rightarrow x \geq 0 \wedge x^2 \leq y \wedge \forall z: int. z^2 \leq y \Rightarrow z \leq x \rangle \rangle \rangle (x+y)(2xu+w)$$

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$$= \langle a: int \rightarrow \langle b: int \rightarrow \langle c: int \rightarrow a \geq 0 \wedge a^2 \leq b \wedge \forall d: int. d^2 \leq b \Rightarrow d \leq a \rangle \rangle \rangle (x+y)(2xu+w)$$

apply 2 times

$$= \langle c: int \rightarrow x+y \geq 0 \wedge (x+y)^2 \leq 2xu+w \wedge \forall d: int. d^2 \leq 2xu+w \Rightarrow d \leq x+y \rangle$$

note that c is unused.

$$= int \rightarrow (x+y \geq 0 \wedge (x+y)^2 \leq 2xu+w \wedge \forall d: int. d^2 \leq 2xu+w \Rightarrow d \leq x+y)$$

(c)

§

$$p(x+z)(y+y)(2+z)$$

$$\langle x: int \rightarrow \langle y: int \rightarrow \langle z: int \rightarrow x \geq 0 \wedge x^2 \leq y \wedge \forall z: int. z^2 \leq y \Rightarrow z \leq x \rangle \rangle \rangle (x+z)(y+y)(2+z)$$

Variables x , y , and z appear both locally and nonlocally.Variable z is introduced twice locally.To avoid confusion, I will rename the local variables to a , b , c , and d .(Since the first local z is unused, I don't need to rename it to c , but I will anyway.)

$$= \langle a: int \rightarrow \langle b: int \rightarrow \langle c: int \rightarrow a \geq 0 \wedge a^2 \leq b \wedge \forall d: int. d^2 \leq b \Rightarrow d \leq a \rangle \rangle \rangle (x+z)(y+y)(2+z)$$

apply 3 times

$$= x+z \geq 0 \wedge (x+z)^2 \leq y+y \wedge \forall d: int. d^2 \leq y+y \Rightarrow d \leq x+z$$