86 Define quantifier $\boxdot$ to give the deep domain of a list or function. Here are some examples.

$\boxdot[[0; 1]; [0; 1]; [0; 1]] \;=\; 0;0,\ 0;1,\ 1;0,\ 1;1,\ 2;0\ \ 2;1$

$\boxdot[10; [11; 12]; 13] \;=\; 0,\ 1;0,\ 1;1,\ 2$

$\boxdot\langle x\text{: } nat\cdot \langle y\text{: } nat\cdot x+y\rangle\rangle \;=\; nat; nat$

$\boxdot\langle x\text{: } 0,..4\cdot \langle y\text{: } 0,..x\cdot x+y\rangle\rangle \;=\; 1;0,\ 2;0,\ 2;1,\ 3;0,\ 3;1,\ 3;2$

After trying the question, scroll down to the solution.

§ To define $\boxdot$ for lists and functions, it is convenient to define it for any x that is not a list or function as

$$\boxdot x \;=\; nil$$

Now we define it for functions:

$$\boxdot v\colon D\cdot\ e \;=\; \langle v\colon D\cdot\ v;\ \boxdot e\rangle\ D$$

Or if you prefer:

$$\boxdot f \;=\; \langle v\colon \square f\cdot\ v;\ \boxdot\ f\ v\rangle\ (\square f)$$

And now for lists:

$$\boxdot L \;=\; \langle n\colon \square L\cdot\ n;\ \boxdot\ L\ n\rangle\ (\square L)$$

We could instead define it in the way we usually define quantifiers.

$$\boxdot v\colon null\cdot\ e \;=\; null$$

$$\boxdot v\colon x\cdot\ e \;=\; x; \langle v\colon x\cdot\ \boxdot e\rangle\ x \qquad \text{for element } x$$

$$\boxdot v\colon A, B\cdot\ e \;=\; (\boxdot v\colon A\cdot\ e), (\boxdot v\colon B\cdot\ e)$$

$$\boxdot v\colon (\S v\colon D\cdot\ b)\cdot\ c \;=\; \langle v\colon D\cdot\ \textbf{if}\ b\ \textbf{then}\ v;\ \boxdot c\ \textbf{else}\ null\ \textbf{fi}\rangle\ D$$