90 (baby) Formalize the statements

Everyone loves my baby.
My baby loves only me.
I am my baby.

and prove that the first two statements imply the last statement.

After trying the question, scroll down to the solution.

§ $\forall p\cdot\ (p\ \mathbf{loves}\ mybaby)$
$(mybaby\ \mathbf{loves}\ me) \land \neg(\exists p\cdot\ p{\neq}me \land (mybaby\ \mathbf{loves}\ p))$
$me{=}mybaby$

Now the proof: starting with the first two statements,

$$(\forall p\cdot\ (p\ \mathbf{loves}\ mybaby)) \land (mybaby\ \mathbf{loves}\ me) \land \neg(\exists p\cdot\ p{\neq}me \land (mybaby\ \mathbf{loves}\ p))$$

duality

$$= \quad (\forall p\cdot\ (p\ \mathbf{loves}\ mybaby)) \land (mybaby\ \mathbf{loves}\ me) \land (\forall p\cdot\ p{=}me \lor \neg(mybaby\ \mathbf{loves}\ p))$$

specialize both ps

$$\Longrightarrow \quad (mybaby\ \mathbf{loves}\ mybaby) \land (mybaby\ \mathbf{loves}\ me)$$
$$\land\ (mybaby{=}me \lor \neg(mybaby\ \mathbf{loves}\ mybaby))$$

use the first conjunct as context

$$= \quad (mybaby\ \mathbf{loves}\ mybaby) \land (mybaby\ \mathbf{loves}\ me) \land (mybaby{=}me \lor \neg\top)$$

binary axiom, base, and specialize

$$\Longrightarrow \quad mybaby{=}me$$

I suspect that "Everyone loves my baby." should be formalized as

$$\forall p\cdot\ p{\neq}mybaby \Rightarrow (p\ \mathbf{loves}\ mybaby)$$

and then the last statement is not provable from the first two.