X2.0 (string arithmetic; research) Define arithmetic (negation, addition, subtraction, multiplication, and division) on strings of real numbers such that if the strings have length 1, they are real arithmetic, and if the strings have length 2, they are complex arithmetic, and if they are longer, the definitions are extended the same way as they are extended from length 1 to length 2.

After trying the question, scroll down to the solution.

§ The string nil will act the same as the number 0. The string $x;y$ will act the same as the complex number $x + i\times y$ where i is the imaginary unit, $i^2 = -1$. Let S and T be strings of the same length, and let x, y, z, and w be real numbers. Then

$-nil \;=\; nil$

$-(S; x) \;=\; -S; -x$

So strings are negated item by item.

$nil + nil \;=\; nil$

$(S; x) + (T; y) \;=\; S{+}T; x{+}y$

So strings are added item by item.

$nil - nil \;=\; nil$

$(S; x) - (T; y) \;=\; S{-}T; x{-}y$

So strings are subtracted item by item.

$nil \times nil \;=\; nil$

$(x; y) \times (z; w) \;=\; x\times z - y\times w;\; y\times z + x\times w$

That gives us complex multiplication, but I don't know how to extend it to longer strings.

$(x; y) \,/\, (z; w) \;=\; (x\times z + y\times w)/(z^2 + w^2); (y\times z - x\times w)/(z^2 + w^2)$

That gives us complex division, but I don't know how to extend it to longer strings.

If strings are of different lengths, we can pad the shorter string with 0s until it is the same length as the longer one. The axiom is $S; 0 \;=\; S$. For example,

$nil; 0 \;=\; nil$ and therefore $0 = nil$

$3; 0 \;=\; 3$ and therefore a complex number with a 0 imaginary part is a real number

$(1; 2) + 3 \;=\; (1; 2) + (3; 0) \;=\; 4; 2$

but this is inconsistent with the definition of string equality that we already have. So we have to modify our definitions of string equality and string order.